Problem 2.3 (same as 1.9) Prove that the probability of occupying an energy level below the Fermi energy equals the probability that an energy level above the Fermi energy and equally far away from the Fermi energy is not occupied.

Solution The probability that an energy level with energy ΔE **below** the Fermi energy E_F is occupied can be rewritten as:

$$f(E_F - \Delta E) = \frac{1}{1 + \exp\dfrac{E_F - \Delta E - E_F}{kT}} = \frac{\exp\dfrac{\Delta E}{kT}}{\exp\dfrac{\Delta E}{kT} + 1}$$

$$= 1 - \frac{1}{\exp\dfrac{\Delta E}{kT} + 1} = 1 - \frac{1}{1 + \exp\dfrac{E_F + \Delta E - E_F}{kT}} = 1 - f(E_F + \Delta E)$$

so that it also equals the probability that an energy level with energy ΔE **above** the Fermi energy, E_F, is **not** occupied.

Problem 2.6 Calculate the effective density of states for electrons and holes in germanium, silicon and gallium arsenide at room temperature and at 100 °C. Use the effective masses for density of states calculations.

Solution The effective density of states in the conduction band for germanium equals:

$$N_c = 2(\frac{2\pi\, m_e^* kT}{h^2})^{3/2}$$

$$= 2(\frac{2\pi\ 0.55 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.626 \times 10^{-34})^2})^{3/2}$$

$$= 1.02 \times 10^{25}\ \text{m}^{-3} = 1.02 \times 10^{19}\ \text{cm}^{-3}$$

where the effective mass for density of states was used (Appendix 3). Similarly one finds the effective densities for silicon and gallium arsenide and those of the valence band, using the effective masses listed below:

	Germanium	Silicon	Gallium Arsenide
m_e/m_0	0.55	1.08	0.067
N_c (cm^{-3})	1.02×10^{19}	2.82×10^{19}	4.35×10^{17}
m_e/m_0	0.37	0.81	0.45
N_v (cm^{-3})	5.64×10^{18}	1.83×10^{19}	7.57×10^{18}

The effective density of states at 100 °C (372.15 K) are obtain from:

$$N_c(T) = N_c(300\,\text{K})(\frac{T}{300})^{3/2}$$

yielding:

T = 100°C	Germanium	Silicon	Gallium Arsenide
N_c (cm^{-3})	1.42×10^{19}	3.91×10^{19}	6.04×10^{17}
N_v (cm^{-3})	7.83×10^{18}	2.54×10^{19}	1.05×10^{18}

Problem 2.7 Calculate the intrinsic carrier density in germanium, silicon and gallium arsenide at room temperature (300 K). Repeat at 100 °C. Assume that the energy bandgap is independent of temperature and use the room temperature values.

Solution The intrinsic carrier density is obtained from:

$$n_i(T) = \sqrt{N_c N_v}\,\exp(\frac{-E_g}{2kT})$$

where both effective densities of states are also temperature dependent. Using the solution of Problem 2.6 one obtains:

T = 300 K	Germanium	Silicon	Gallium Arsenide
n_i (cm^{-3})	2.16×10^{13}	8.81×10^{9}	1.97×10^{6}
T = 100°C	Germanium	Silicon	Gallium Arsenide
n_i (cm^{-3})	3.67×10^{14}	8.55×10^{11}	6.04×10^{8}

Problem 2.13 The resistivity of a silicon wafer at room temperature is 5 Ωcm. What is the doping density? Find all possible solutions.

Solution Starting with a initial guess that the conductivity is due to electrons with a mobility of 1400 cm^2/V-s, the corresponding doping density equals:

$$N_d \cong n = \frac{1}{q\mu_n\rho} = \frac{1}{1.6\times10^{-19}\times1400\times5} = 8.9 \text{ x } 10^{14} \text{ cm}^{-3}$$

The mobility corresponding to this doping density equals

$$\mu_n = \mu_{\min} + \frac{\mu_{\max} - \mu_{\min}}{1 + (\frac{N_d}{N_r})^{\alpha}} = 1366 \text{ cm}^2\text{/V-s}$$

Since the calculated mobility is not the same as the initial guess, this process must be repeated until the assumed mobility is the same as the mobility corresponding to the calculated doping density, yielding:

$$N_d = 9.12 \text{ x } 10^{14} \text{ cm}^{-3} \text{ and } \mu_n = 1365 \text{ cm}^2\text{/V-s}$$

For p-type material one finds:

$$N_a = 2.56 \text{ x } 10^{15} \text{ cm}^{-3} \text{ and } \mu_p = 453 \text{ cm}^2\text{/V-s}$$

Problem 2.16 A piece of intrinsic silicon is instantaneously heated from 0 K to room temperature (300 K). The minority carrier lifetime due to Shockley-Read-Hall recombination in the material is 1 ms. Calculate the generation rate of electron-hole pairs immediately after reaching room temperature. ($E_t = E_i$). If the generation rate is constant, how long does it take to reach thermal equilibrium?

Solution As the material is instantaneously heated from 0 K, the initial carrier densities will still be the same as at 0 K, namely zero.
The initial generation rate will then be:

$$G_{p,SHR} = G_{n,SHR} = -\frac{0 - n_i^2}{0 + 0 + 2n_i}\frac{1}{\tau} = \frac{n_i}{2\tau} = 5 \text{ x } 10^{12} \text{ cm}^{-3}\text{s}^{-1}$$

Where n_i is the intrinsic carrier density at room temperature. If this generation rate were to continue until the electron and hole densities equal the intrinsic carrier density at room temperature, the time needed would be:

$$t = \frac{n_i}{G_{p,SHR}} = 2\tau = 2 \text{ ms}$$

Problem 2.22 Calculate the probability that an electron occupies an energy level, which is $3kT$ below the Fermi energy. Repeat for an energy level, which is $3kT$ above the Fermi energy.

Solution The probability that an electron occupies an energy level, which is $3kT$ **below** the Fermi energy equals:

$$f(E_F - 3kT) = \frac{1}{1 + \exp(\frac{E_F - 3kT - E_F}{kT})} = \frac{1}{1 + \exp(-3)} = 95.3\ \%$$

The probability that an electron occupies an energy level, which is $3kT$ **above** the Fermi energy equals:

$$f(E_F + 3kT) = \frac{1}{1 + \exp(\frac{E_F + 3kT - E_F}{kT})} = \frac{1}{1 + \exp(3)} = 4.7\ \%$$

Problem 2.25 Electrons in silicon carbide have a mobility of 1000 cm^2/V-sec. At what value of the electric field do the electrons reach a velocity of 3 x 10^7 cm/s? Assume that the mobility is constant and independent of the electric field. What voltage is required to obtain this field in a 5 micron thick region? How much time do the electrons need to cross the 5 micron thick region?

Solution The electric field is obtained from the mobility and the velocity:

$$E = \frac{\mu}{v} = \frac{1400}{3 \times 10^7} = 30 \text{ kV/cm}$$

Combined with the length one finds the applied voltage.

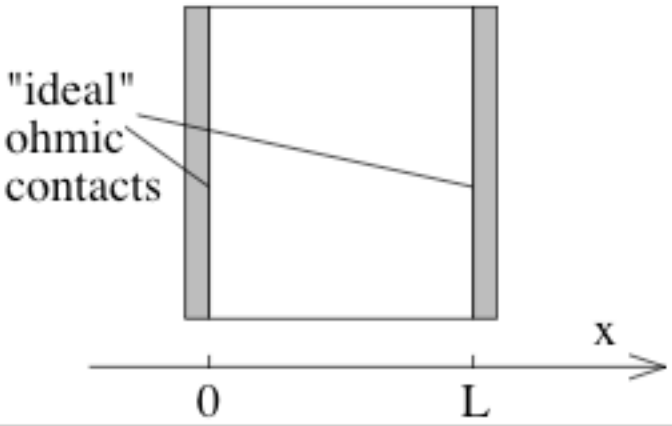
$$V = E\, L = 30{,}000 \text{ x } 5 \text{ x } 10^{-4} = 15 \text{ V}$$

The transit time equals the length divided by the velocity:

$$t_r = L/v = 5 \text{ x } 10^{-4}/3 \text{ x } 10^{7} = 16.7 \text{ ps}$$

Problem 2.33

An intrinsic piece of GaAs ($n_i = 2 \times 10^6$ cm^{-3}) of length L (= 1 μm) is uniformly illuminated with light yielding an electron-hole pair generation rate of 10^{22} cm^{-3} s^{-1}. On both ends (at x = 0 and $x = L$) the material is contacted with "ideal" ohmic contacts. Calculate the maximum value of the steady-state excess hole density in the material under illumination. The hole diffusion length, L_p, equals $\frac{L}{2\ln 2}$ and the hole mobility is 500 cm^2/V-s. Assume that there is no electric field in the semiconductor and T = 300 K.



Solution

The diffusion equation under uniform illumination equals:

$$0 = D_p \frac{d^2\delta p(x)}{dx^2} - \frac{\delta p(x)}{\tau_p} + G_{op} = \frac{d^2\delta p(x)}{dx^2} - \frac{\delta p(x)}{L_p^2} + \frac{G_{op}}{D_p}$$

resulting in:

$$\delta p(x) = G_{op}\tau_p[1 - \frac{\cosh(\frac{2x - L}{2L_p})}{\cosh(\frac{L}{2L_p})}]$$

The maximum occurs at L/2 yielding:

$$\delta p(x) = G_{op}\tau_p[1 - \frac{1}{\cosh(\ln 2)}] = \frac{G_{op}\tau_p}{5} = 8.05 \text{ x } 10^{11} \text{ cm}^{-3}$$

Where the hole life time is obtained from:

$$\tau_p = \frac{L_p}{\mu_p V_t} = \frac{L}{\mu_p V_t 2\ln 2} = 0.4 \text{ ns}$$