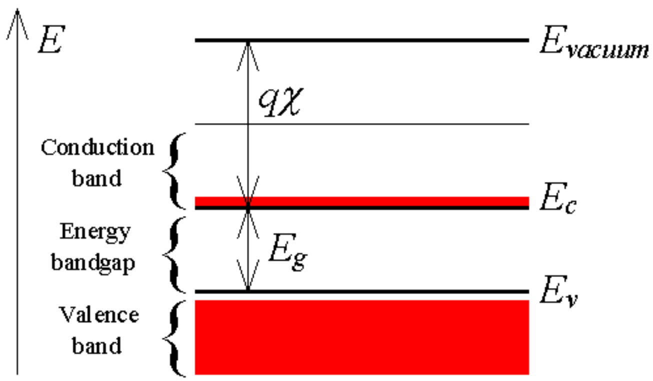
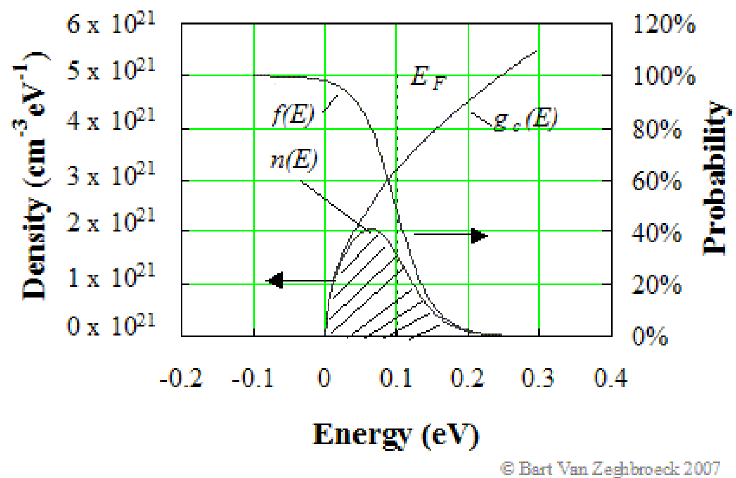
Review

Energy Bands



Carrier Density & Mobility



Carrier Transport

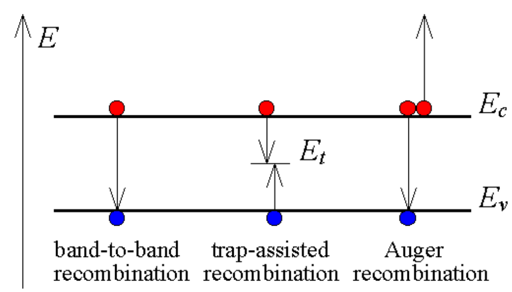
$$J_n = q n \mu_n \mathcal{E} + q D_n \frac{dn}{dx}$$

$$J_p = -q D_p \frac{dp}{dx}$$

$$\mu = \frac{q \tau_c}{m^*}$$

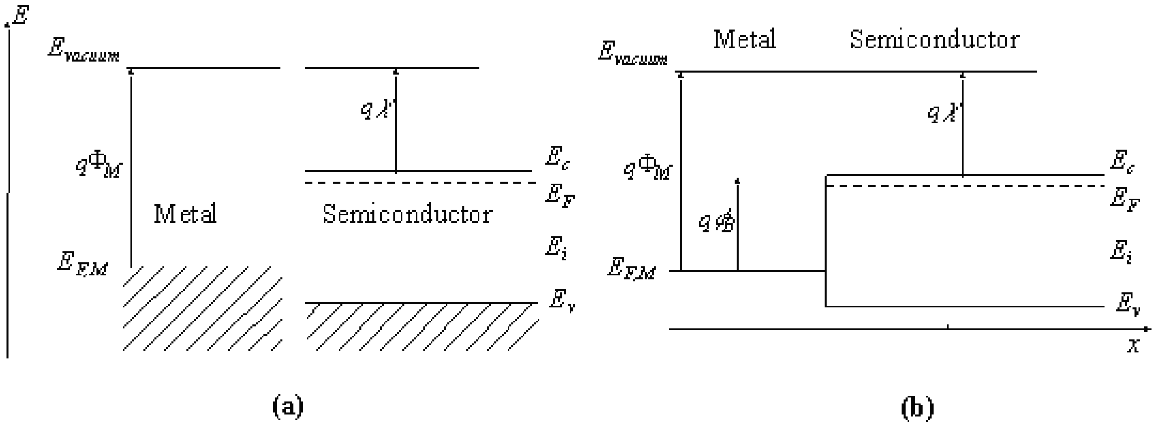
$$\sigma = \frac{\Delta J}{\mathcal{E}} = q(n\mu_n + p\mu_p)$$

Generation and Recombination



$$G_{p,light} = G_{n,light} = \alpha \frac{P_{opt}(x)}{E_{ph} A}$$

The Metal-Semiconductor Junction: Review



Energy band diagram of the metal and the semiconductor before (a) and after (b) contact

Barrier Height

$$\phi_B = \Phi_M - \chi \quad \text{, for an n - type semiconductor}$$

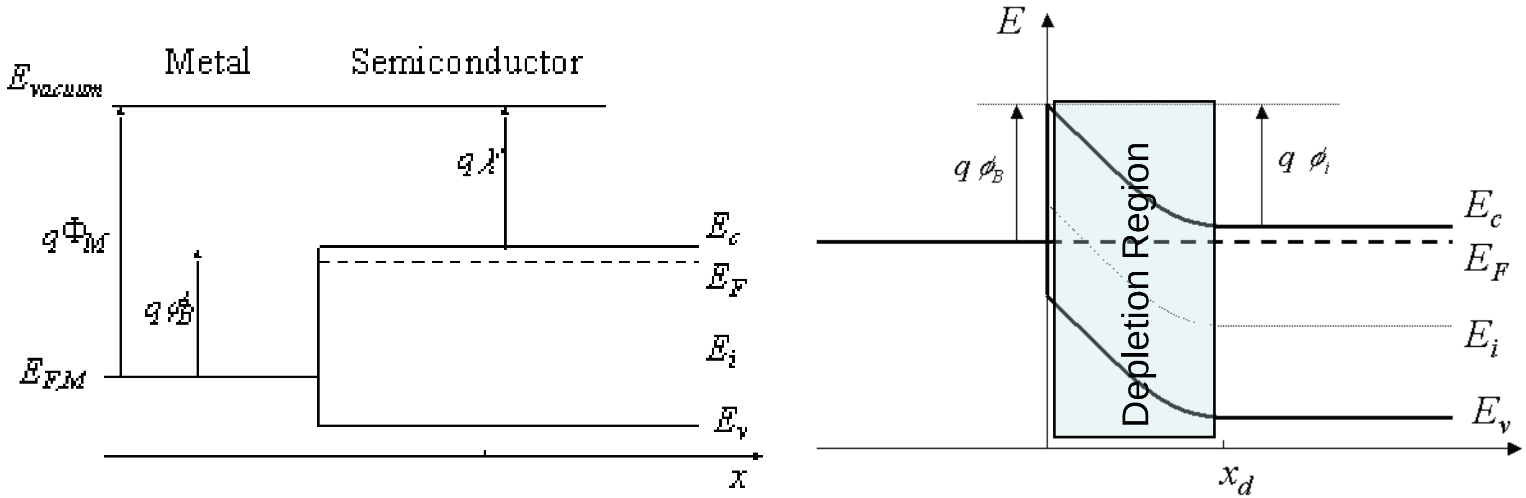
$$\phi_B = \frac{E_g}{q} + \chi - \Phi_M \quad \text{, for a p - type semiconductor}$$

Built-in Potential

$$\phi_i = \Phi_M - \chi - \frac{E_c - E_{F,n}}{q} \quad , \quad \text{n - type}$$

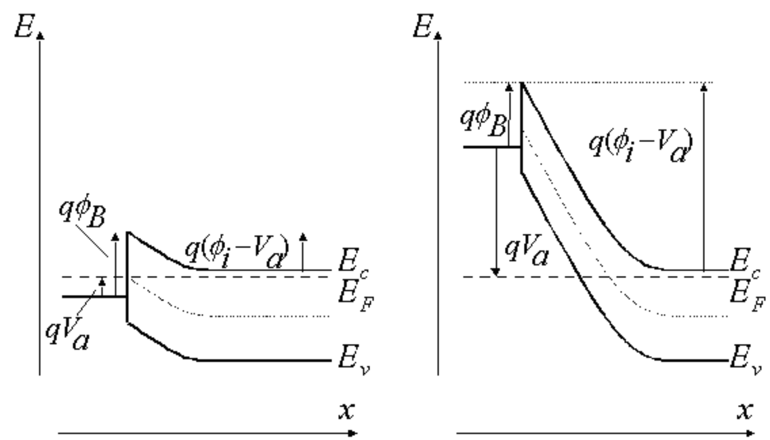
$$\phi_i = \chi + \frac{E_c - E_{F,p}}{q} - \Phi_M \quad , \quad \text{p - type}$$

M-S Junctions: Thermal Equilibrium



Energy band diagram of a metal-semiconductor contact in thermal equilibrium.

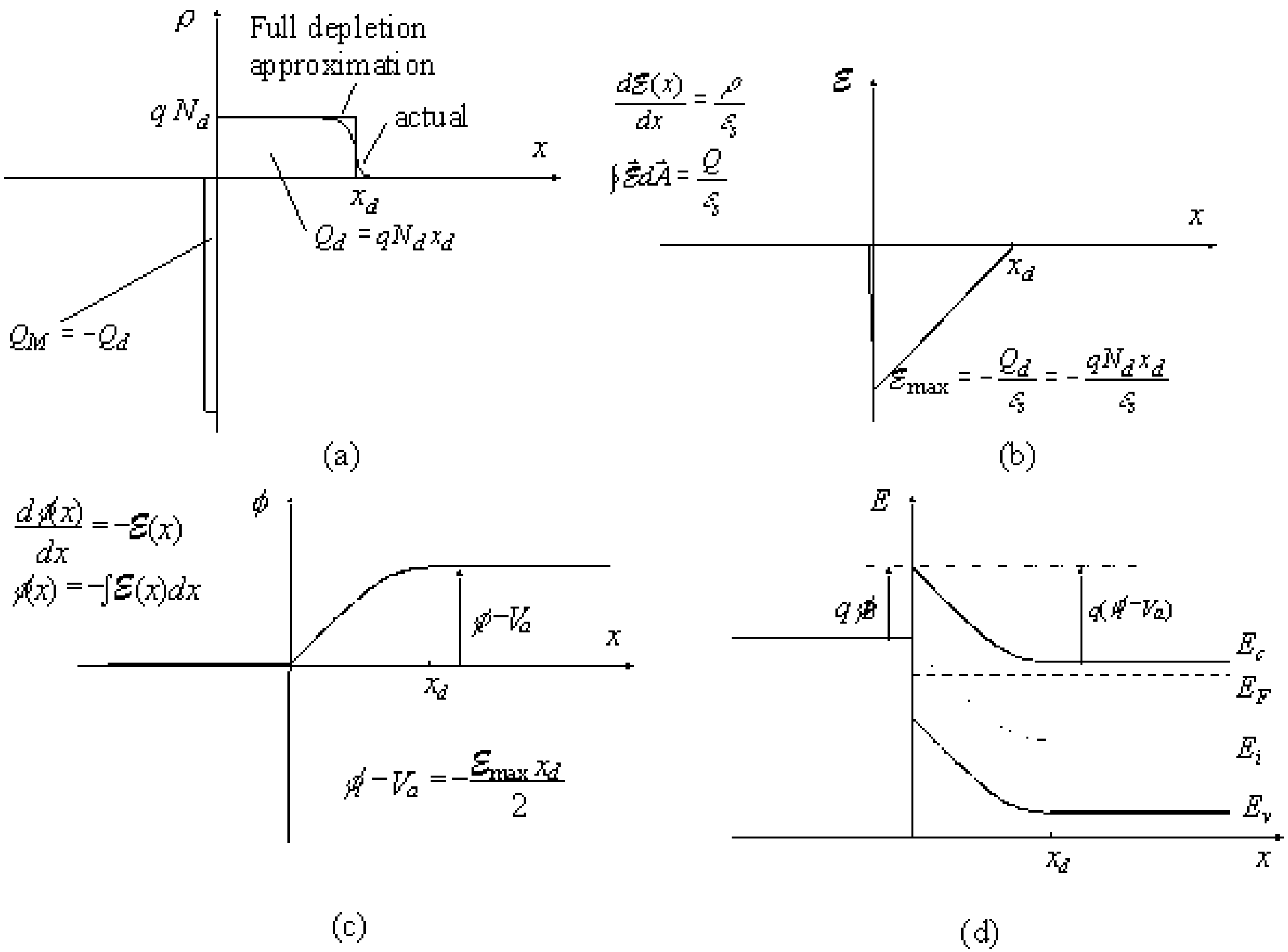
Under Bias



(a) (b)

$$\phi(x=\infty) - \phi(x=0) = \phi_i - V_a$$

Full depletion approximation Review



$$\phi_i - V_a = -\phi(x=0) = \frac{qN_d x_d^2}{2\varepsilon_s}$$

$$x_d = \sqrt{\frac{2\varepsilon_s(\phi_i - V_a)}{qN_d}}$$

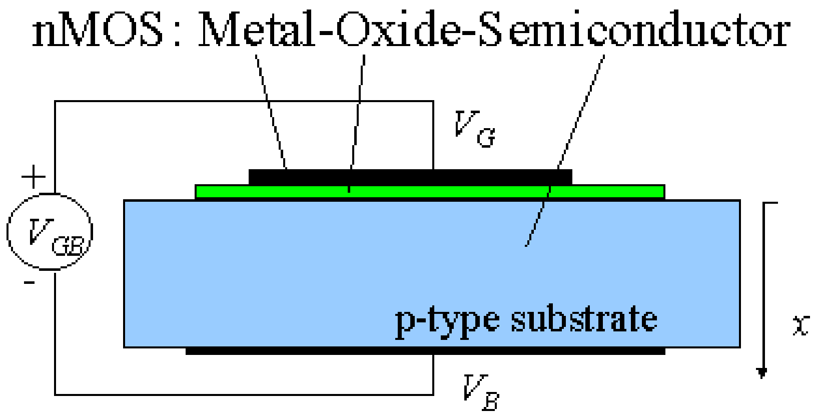
$$C_j = \left|\frac{dQ_d}{dV_a}\right| = \sqrt{\frac{q\varepsilon_s N_d}{2(\phi_i - V_a)}} = \frac{\varepsilon_s}{x_d}$$

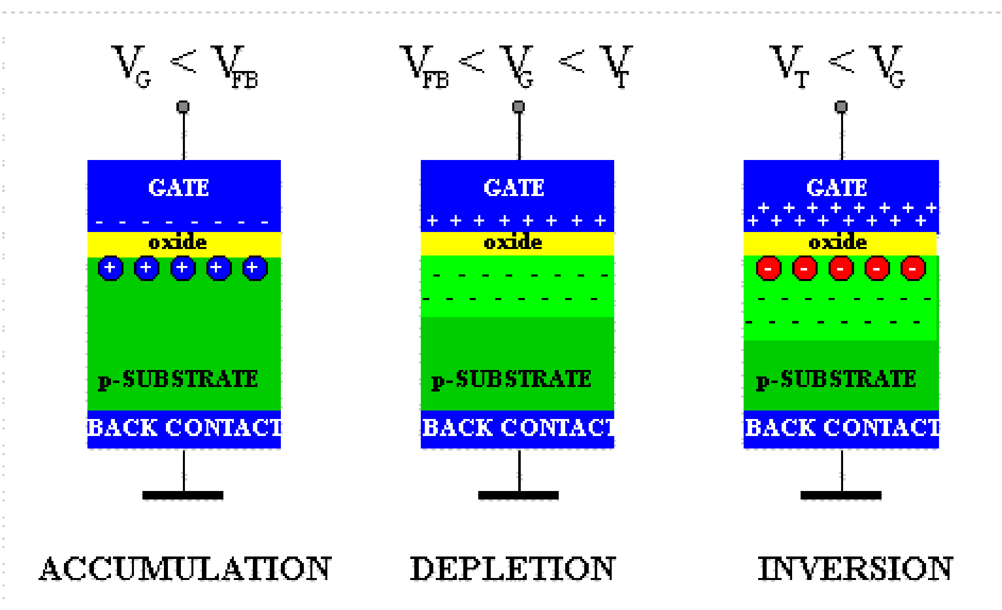
$$\Delta\phi_B = \sqrt{\frac{q\mathcal{E}_{max}}{4\pi\varepsilon_s}}$$

a) Charge density, (b) electric field, (c) potential and (d) energy with the full depletion analysis.

MOS Capacitors: Review

The MOS capacitor consists of a Metal-Oxide-Semiconductor structure



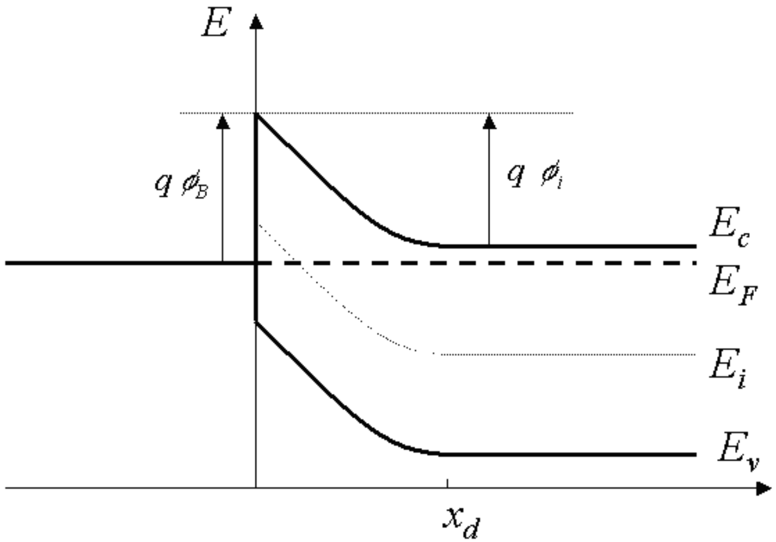


$$V_{FB} = \Phi_M - \Phi_S$$

$$\Phi_M - \Phi_S = \Phi_M - \chi - \frac{E_g}{2q} - V_t \ln\left(\frac{N_a}{n_i}\right)$$

Current Transport at the MS Interface

The current across a metal-semiconductor junction is mainly due to majority carriers. Three distinctly different mechanisms exist: diffusion of carriers from the semiconductor into the metal, thermionic emission of carriers across the Schottky barrier and quantum-mechanical tunneling through the barrier.



Diffusion Current: driving force is distributed over the length of the depletion layer.

Thermionic Emission: only energetic carriers, with energy equal to or larger than the conduction band energy at the metal-semiconductor interface, contribute to the current flow.

Tunneling: the wave-nature of the electrons, allowing them to penetrate through thin barriers.

Diffusion, Thermionic Emission & Tunneling

For Diffusion current, the depletion layer is large compared to the mean free path, so that the concepts of drift and diffusion are valid. The current depends exponentially on the applied voltage, V_a, and the barrier height, ϕ_B.

$$J_n = \frac{q^2 D_n N_c}{V_t} \sqrt{\frac{2q(\phi_i - V_a) N_d}{\varepsilon_s}} \exp\left(-\frac{\phi_B}{V_t}\right)\left[\exp\left(\frac{V_a}{V_t}\right) - 1\right]$$

Electric-field at MS Junction: $\mathcal{E}_{max} = \sqrt{\frac{2q(\phi_i - V_a) N_d}{\varepsilon_s}}$

The thermionic emission theory assumes that electrons, with an energy larger than the top of the barrier, will cross the barrier provided they move towards the barrier. The actual shape of the barrier is ignored.

$$J_{MS} = A^* T^2 e^{-\phi_B / V_t} (e^{V_a / V_t} - 1)$$

ϕ_B is the Schottky barrier height.

Richardson's Constant $A^* = \frac{4 \pi q m^* k^2}{h^3}$ and velocity $v_R = \sqrt{\frac{kT}{2 \pi m}}$

For tunneling, the carrier density equals the density of available electrons, n, multiplied with the tunneling probability, Θ, yielding:

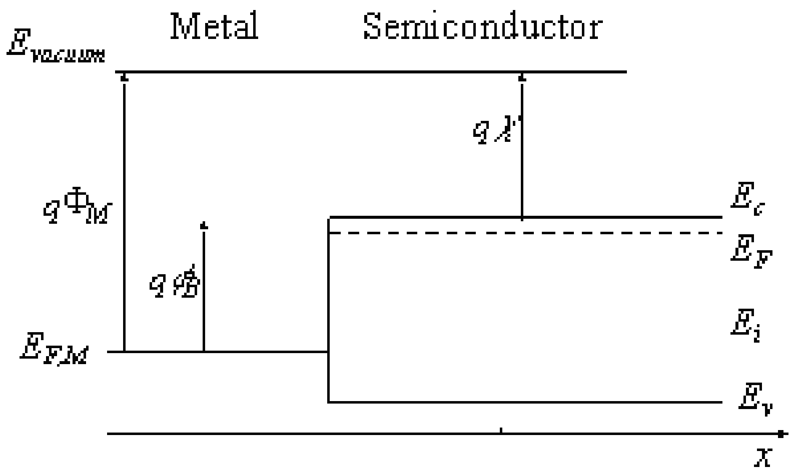
$$J_n = q v_R n \Theta$$

$$\Theta = \exp\left(-\frac{4}{3} \frac{\sqrt{2qm^*}}{\hbar} \frac{\phi_B^{3/2}}{\mathcal{E}}\right)$$

where $\varepsilon = \phi_B / L$

Metal-Semiconductor Contacts

MS contacts cannot be assumed to have a resistance as low as that of two connected metals. In particular, a large mismatch between the Fermi energy of the metal and semiconductor can result is a high-resistance rectifying contact. Low resistance Ohmic contacts or tunnel contacts can be created.



Ohmic contacts exist if the Schottky barrier height, ϕ_B, is zero or negative. For an n-type semiconductor, ϕ_M is close or smaller than χ. For a p-type semiconductor, ϕ_M is close or greater than $\chi + E_g$

Tunnel contacts have a positive barrier at the MS interface, but also have a high enough doping in the semiconductor that there is only a thin barrier separating the metal from the semiconductor. If the width of the depletion region at the MS interface is very thin, on the order of 3 nm or less, carriers can readily tunnel across such barrier. The typical required doping density for such contact is 10^{19} cm^{-3} or higher.

Different methods of forming Contacts

Ohmic:

A high temperature step is used so metal can either alloy with the semiconductor or reduce the unintentional barrier at the interface.

n and p-doping are frequently used as a method to form Ohmic contacts.

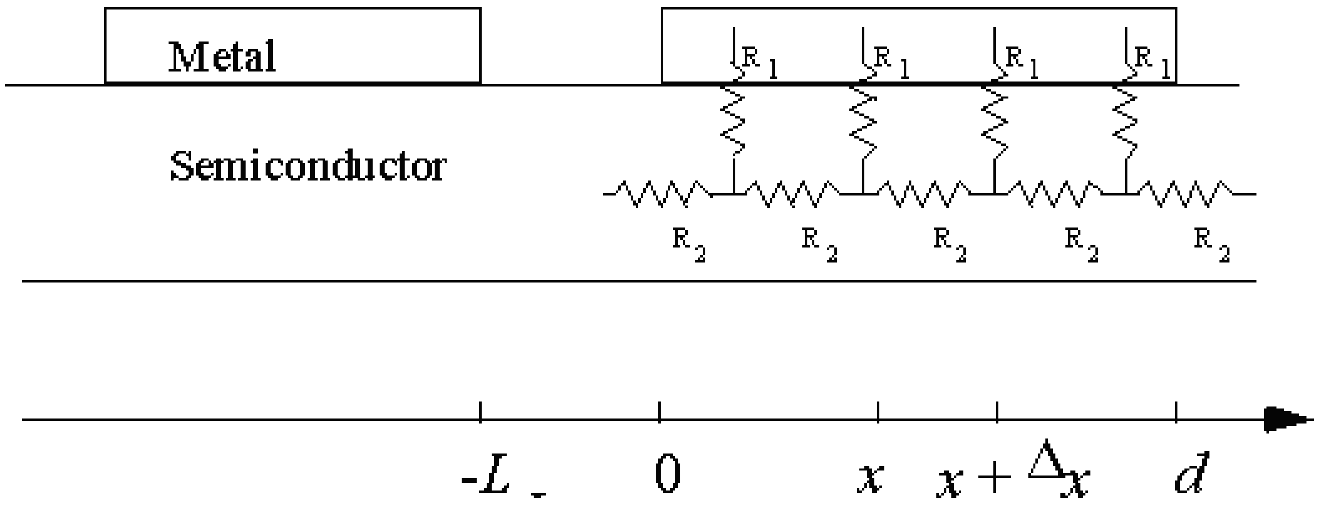
Tunneling Contacts:

Contacts deposited at Low-T tend to form Shottky barriers, or tunneling, contacts

Pinning of the Fermi energy at the interface due to the large number of surface states at the metal-semiconductor interface can also occur, leading to a tunneling contact.

Contact Resistance

This equivalent circuit is obtained by slicing the structure into small sections with length Δx.



The contact resistance, R_1, and the semiconductor resistance, R_2, are given by:

$$R_1 = \frac{\rho_c}{W\Delta x} \qquad R_2 = R_s \frac{\Delta x}{W}$$

where ρ_c is the contact resistance of the metal-to-semiconductor interface per unit area with units of Ωcm^2, R_s is the sheet resistance of the semiconductor layer with units of Ohms per square, and W is the width of the contact.

Contact Resistance

Kirchoff's Laws: $V(x+\Delta x) - V(x) = I(x)R_2 = I(x)\dfrac{R_s}{W}\Delta x$ $\quad I(x+\Delta x) - I(x) = \dfrac{V(x)}{R_1} = V(x)\dfrac{W}{\rho_c}\Delta x$

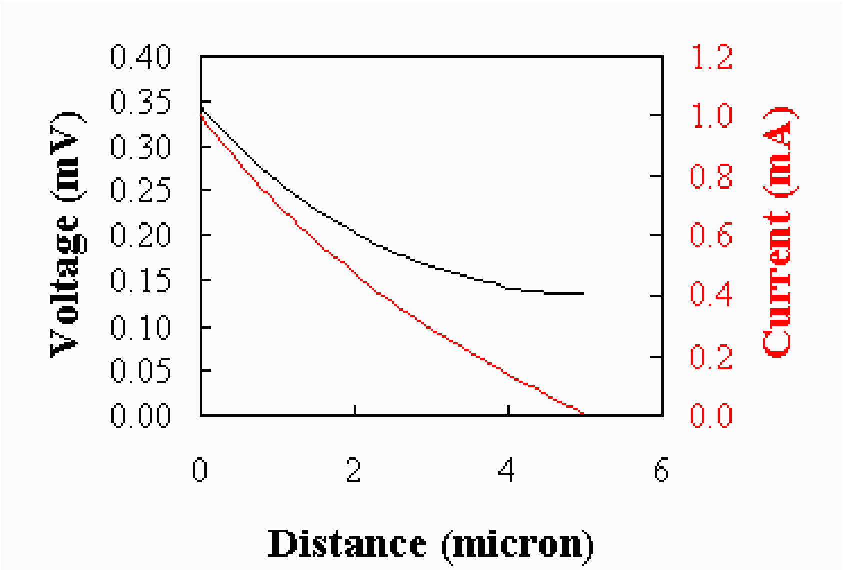
Letting Δx go to 0: $\dfrac{dV}{dx} = \dfrac{I(x)R_s}{W}$ $\quad \dfrac{dI}{dx} = \dfrac{V(x)W}{\rho_c}$

$$\frac{d^2 I(x)}{dx^2} = I(x)\frac{R_s}{\rho_c} = \frac{I(x)}{\lambda^2} \text{ with } \lambda = \sqrt{\frac{\rho_c}{R_s}}$$

λ is the characteristic distance over which the current changes under the metal contact and is also referred to as the penetration length.

$$I(x) = I_0 \frac{\sinh \frac{d-x}{\lambda}}{\sinh \frac{d}{\lambda}}$$

$$V(x) = I_0 \frac{\lambda R_s}{W} \frac{\cosh \frac{d-x}{\lambda}}{\sinh \frac{d}{\lambda}}$$



$$R_c = \frac{V(0)}{I(0)} = \frac{\lambda R_s}{W} \coth \frac{d}{\lambda} = \frac{\sqrt{\rho_c R_s}}{W} \coth \frac{d}{\lambda}$$

Lateral current and voltage underneath a 5 mm long and 1 mm wide metal contact with a ρ_c of 10^{-5} Ω-cm^2 on a thin semiconductor layer with a R_s of 100 Ohms per sq.

Currents Through Insulators

Current mechanisms through materials which do not contain free carriers (i.e. insulators) can be distinctly different from those in doped semiconductors or metals.

Fowler-Nordheim Tunneling: carriers quantum mechanically tunnel from the conductor into the insulator.

Poole-Frenkel emission: Traps restrict current flow because of a capture and emission process

Space charge Limited Current: A high density of charged carriers causes a field gradient, which limits the current density.

Ballistic transport: Carrier transport without scattering or any other mechanism, which would cause a loss of energy.

Fowler-Nordheim Tunneling

Fowler-Nordheim tunneling has been studied extensively in Metal-Oxide-Semiconductor structures where it is the dominant current mechanism, especially for thick oxides.

Quantum mechanical tunneling from the adjacent conductor into the insulator limits the current through the structure. Once the carriers have tunneled into the insulator they are free to move within the valence or conduction band of the insulator. The calculation of the current is based on the WKB approximation yielding the following relation between the current density, J_{FN}, and the electric field in the oxide, ε_{ox}:

$$J_{FN} = C_{FN}\varepsilon_{ox}^2 \exp_{ox}\left(-\frac{4}{3}\frac{\sqrt{2m_{ox}^*}}{q\hbar}\frac{(q\phi_B)^{3/2}}{\varepsilon_{ox}}\right)$$

For tunneling

$$\varepsilon_{ox} d \geq \phi_B$$

To check for this current mechanism, experimental I-V characteristics are typically plotted as $\ln(J_{FN} / \varepsilon_{ox})$ versus $1/\varepsilon_{ox}$, a so-called Fowler-Nordheim plot. Provided the effective mass of the insulator is known, one can then fit the experimental data to a straight line yielding a value for the barrier height.

Poole-Frenkel Emission

Structural defects can cause additional energy states close to the band edge, called traps. These traps restrict the current flow because of a capture and emission process, thereby becoming the dominant current mechanism. The current is a simple drift current described by:

$$J = qn\mu \mathcal{E}_N$$

while the carrier density depends exponentially on the depth of the trap, which is corrected for the electric field

$$n = n_0 \exp[-\frac{q}{kT}(\phi_B - \sqrt{\frac{q\mathcal{E}_N}{\pi\varepsilon_N}})]$$

The total current then equals:

$$J_{PF} = qn_0\mu\mathcal{E}_N \exp[-\frac{q}{kT}(\phi_B - \sqrt{\frac{q\mathcal{E}_N}{\pi\varepsilon_N}})]$$

Space Charge Limited Current

Both Fowler-Nordheim tunneling and Poole-Frenkel emission mechanism yield very low current densities with correspondingly low carrier densities. For structures where carriers can readily enter the insulator and freely flow through the insulator one finds that the resulting current and carrier densities are much higher. The high density of these charged carriers causes a field gradient, which limits the current density.

$$J = qp\mu\mathcal{E} \qquad \frac{d\mathcal{E}}{dx} = \frac{qp}{\varepsilon}$$

Some math (see HW problem): $$J = \frac{9\varepsilon_i\mu V^2}{8d^3}$$

Ballistic Transport

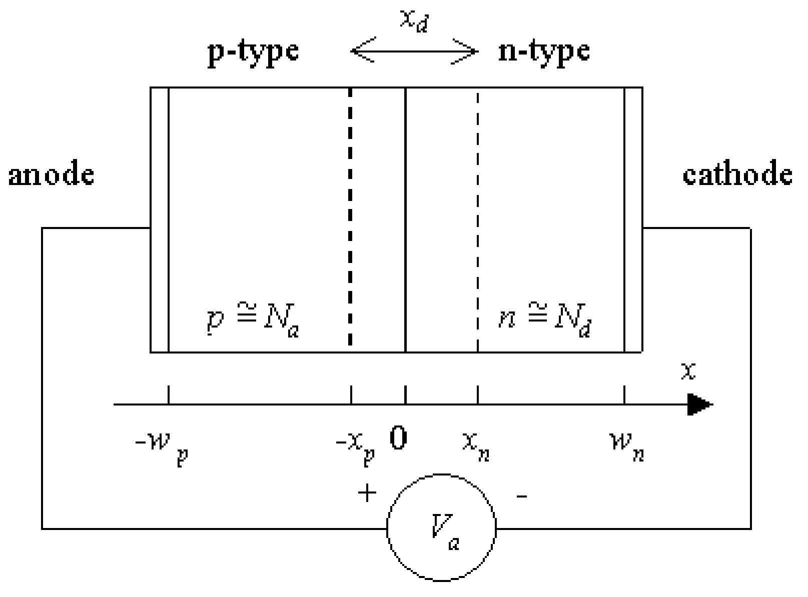
Ballistic transport is carrier transport without scattering or any other mechanism, which would cause a loss of energy. Combining energy conservation, current continuity and Gauss's law one finds the following current-voltage relation:

$$J = \frac{4\varepsilon}{9}\sqrt{\frac{2q}{m^*}}\frac{V^{3/2}}{d^2}$$

where d is the thickness of the insulator and m* is the effective mass of the carriers.

The p-n Junction

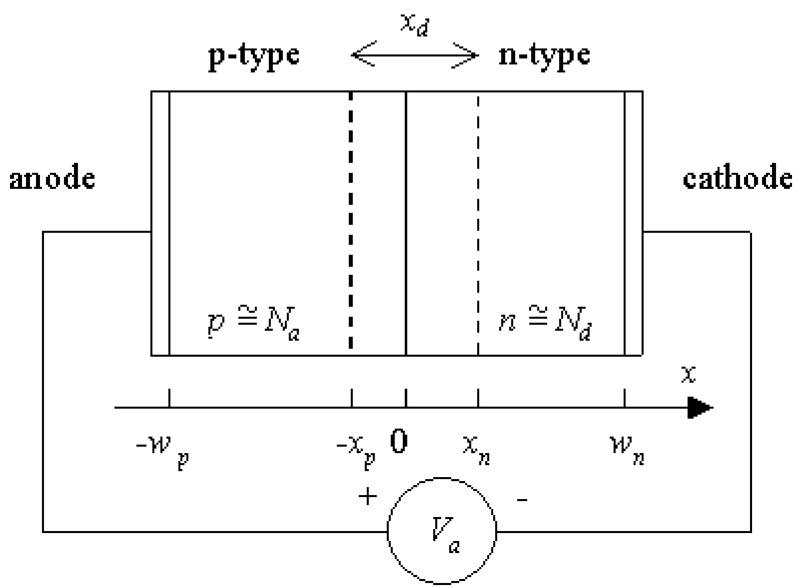
A p-n junction consists of two semiconductor regions with opposite doping type. The region on the left is p-type with an acceptor density N_a, and on the right is n-type with a donor density N_d. The dopants are assumed to be shallow, so that the electron (hole) density in the n-type (p-type) region is approximately equal to the donor (acceptor) density.



Abrupt p-n junction: the doped regions are uniformly doped sp that the transition between the two regions is abrupt.

One-sided abrupt p-n junction: one side of the p-n junctions in is distinctly higher-doped than the other in which case only the low-doped region needs to be considered.

The p-n Junction under bias



The junction is biased with a voltage V_a.

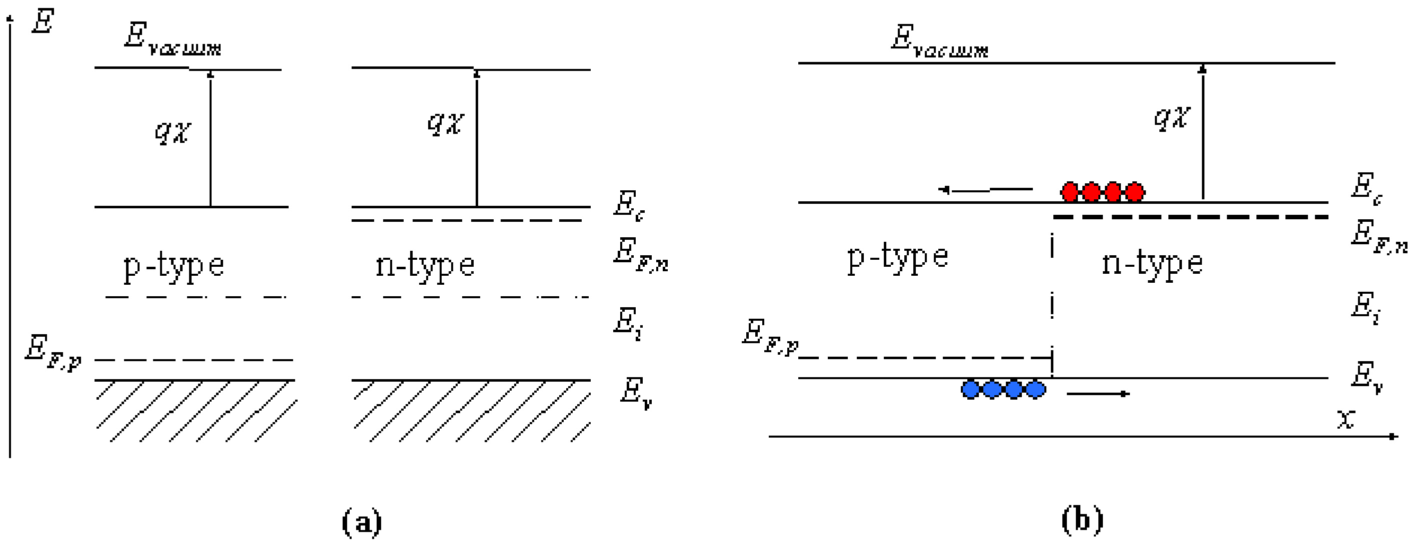
Forward-biased: a positive voltage is applied to the p-doped region

Reversed-biased: a negative voltage is applied to the p-doped region.

Anode: the p-type region

Cathode: the n-type region

The p-n Junction: Flat Band Diagram



Energy band diagram of a p-n junction (a) before and (b) after merging the n-type and p-type regions

The flatband diagram is not an equilibrium diagram since both electrons and holes can lower their energy by crossing the junction. The flatband diagram implies that there is no field and no net charge in the semiconductor.

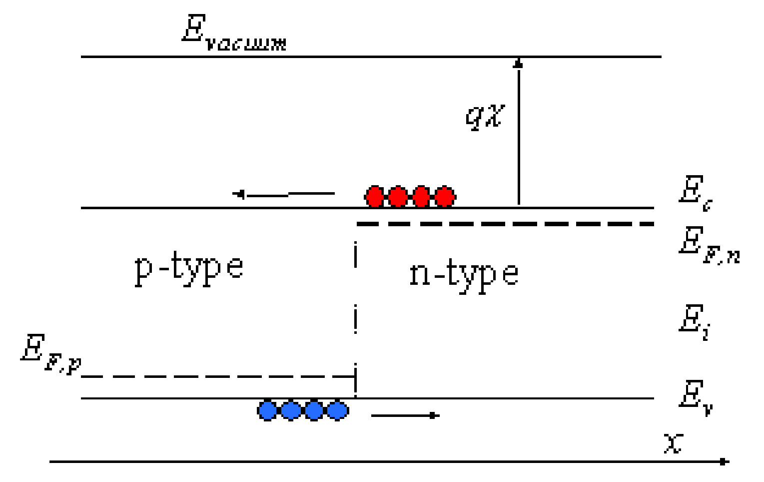
The p-n Junction: Thermal Equilibrium

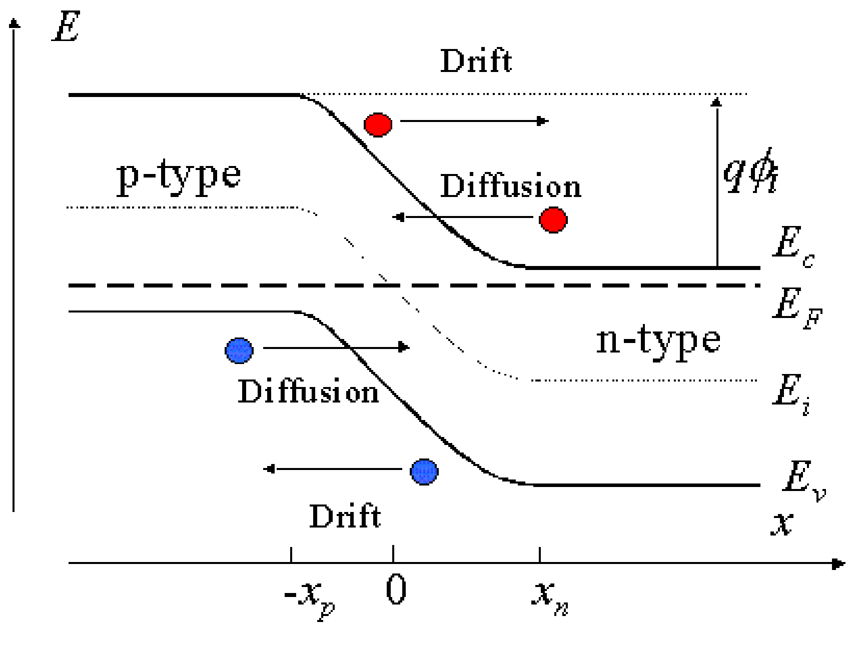
• To reach thermal equilibrium, electrons/holes diffuse across the junction into the p-type/n-type region.

• This process leaves the ionized donors (acceptors) behind, creating a region around the junction, which is depleted of mobile carriers. The depletion region extends from $x = -x_p$ to $x = x_n$.

• The charge due to the ionized donors and acceptors causes an electric field, which in turn causes a drift of carriers in the opposite direction.

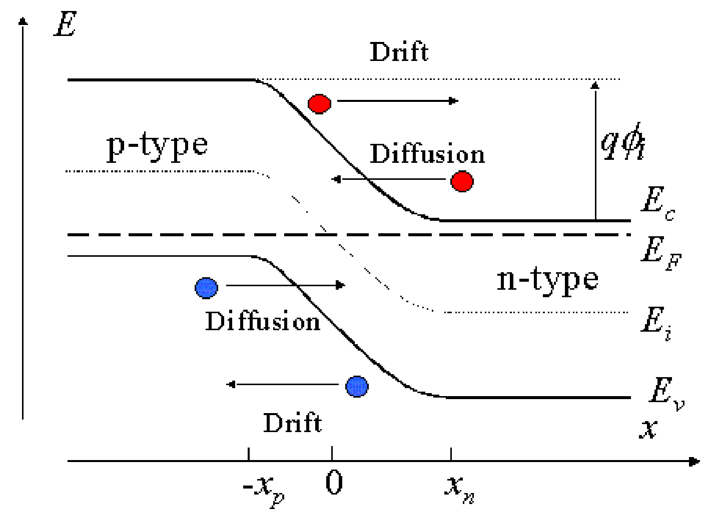
• The diffusion of carriers continues until the drift current balances the diffusion current, thereby reaching thermal equilibrium as indicated by a constant Fermi energy.





The p-n Junction: Built-in Potential

While in thermal equilibrium no external voltage is applied between the n-type and p-type material, there is an internal potential, ϕ_i, or built-in potential, which is caused by the workfunction difference between the n-type and p-type semiconductors.



The built-in potential in a semiconductor equals the potential across the depletion region in thermal equilibrium. Since thermal equilibrium implies that the Fermi energy is constant throughout the p-n diode, the built-in potential equals the difference between the Fermi energies, E_{Fn} and E_{Fp}, divided by the electronic charge.

$$\phi_i = V_t \ln \frac{N_d N_a}{n_i^2}$$

It also equals the sum of the bulk potentials of each region, ϕ_n and ϕ_p, since the bulk potential quantifies the distance between the Fermi energy and the intrinsic energy.

The p-n Junction: Built-in Potential

An abrupt silicon p-n junction consists of a p-type region containing 2 x 10^{16} cm^{-3} acceptors and an n-type region containing also 10^{16} cm^{-3} acceptors in addition to 10^{17} cm^{-3} donors.

1. Calculate the thermal equilibrium density of electrons and holes in the p-type region as well as both densities in the n-type region.
2. Calculate the built-in potential of the p-n junction

Solution

1. The thermal equilibrium densities are:

In the p-type region:

$p = N_a = 2 \times 10^{16}\ cm^{-3}$

$n = n_i^2/p = 10^{20}/2 \times 10^{16} = 5 \times 10^3\ cm^{-3}$

In the n-type region

$n = N_d - N_a = 9 \times 10^{16}\ cm^{-3}$

$p = n_i^2/n = 10^{20}/(1 \times 10^{16}) = 1.11 \times 10^3\ cm^{-3}$

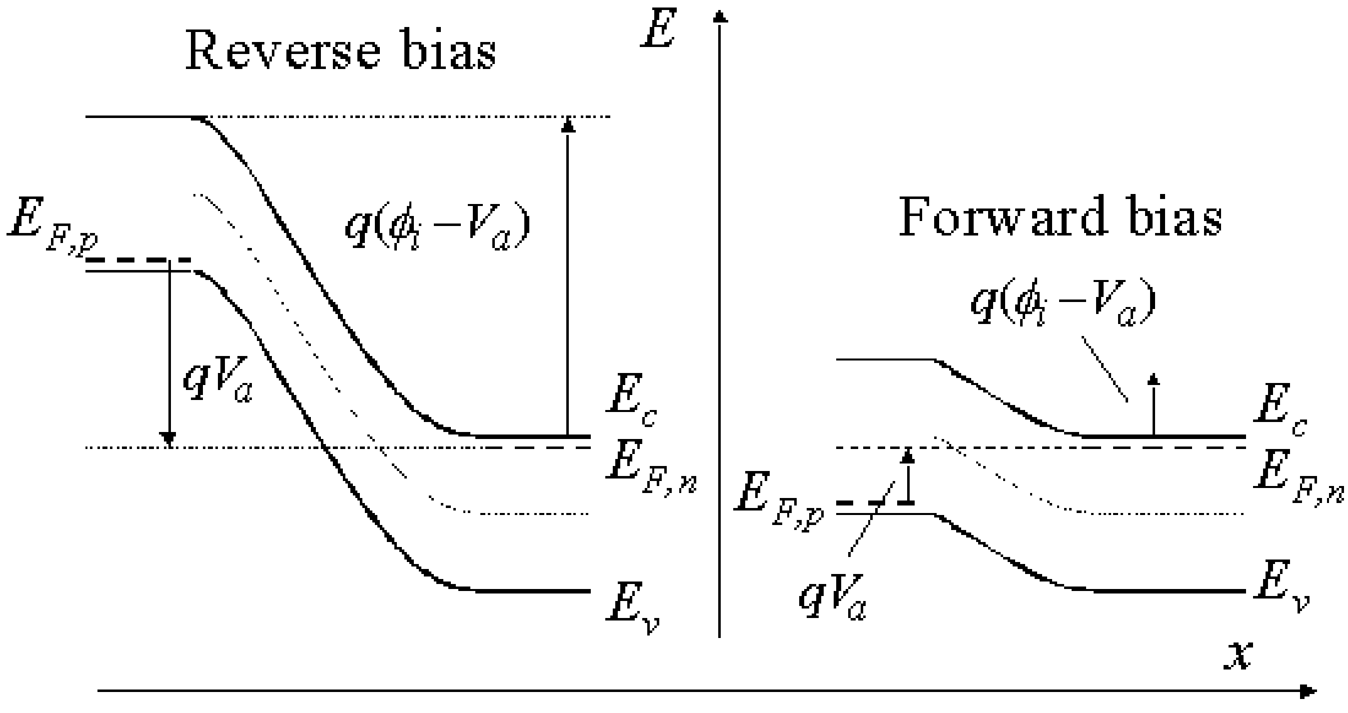
2. The built-in potential is obtained from:

$$\phi_i = V_t \ln \frac{p_n n_p}{n_i^2} = 0.0259 \ln \frac{2 \times 10^{16} \times 9 \times 10^{16}}{10^{20}} = 0.79\ \text{V}$$

The p-n Junction: Forward and Reverse Bias

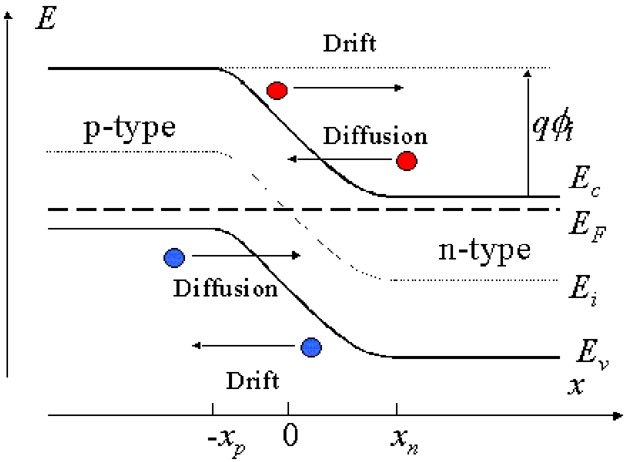
As a negative voltage is applied, the potential across the semiconductor increases and so does the depletion layer width. As a positive voltage is applied, the potential across the semiconductor decreases and with it the depletion layer width. The total potential across the semiconductor equals the built-in potential minus the applied voltage, or:

$$\phi = \phi_i - V_a$$



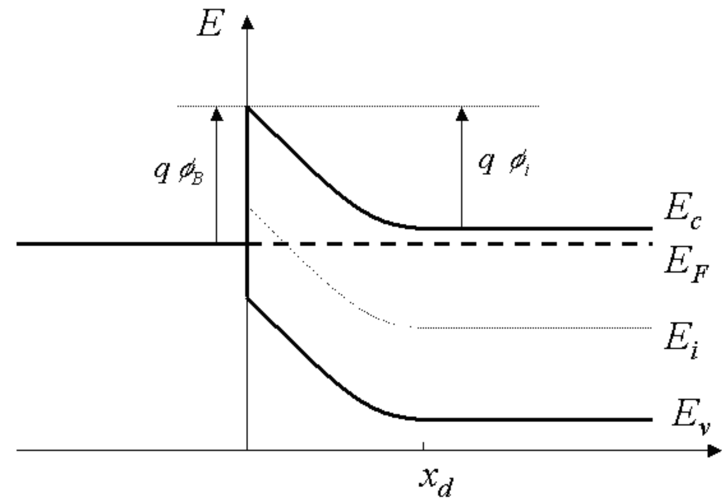
Energy band diagram of a p-n junction under reverse and forward bias

The p-n Junction vs MS Junction



$$\phi_i = V_t \ln \frac{N_d N_a}{n_i^2}$$

$$\phi = \phi_i - V_a$$



$$\phi_B = \Phi_M - \chi \quad \text{, for an n-type semiconductor}$$

$$\phi_B = \frac{E_g}{q} + \chi - \Phi_M \quad \text{, for a p-type semiconductor}$$

$$\phi_i = \Phi_M - \chi - \frac{E_c - E_{F,n}}{q} \quad , \quad \text{n-type}$$

$$\phi_i = \chi + \frac{E_c - E_{F,p}}{q} - \Phi_M \quad , \quad \text{p-type}$$

$$\phi_i = \phi_B - V_t \ln \frac{N_c}{N_d}$$

$$\phi(x = \infty) - \phi(x = 0) = \phi_i - V_a$$

The p-n Junction: Poisson's Equation

The electrostatic analysis of a p-n diode is of interest since it provides knowledge about the charge density and the electric field in the depletion region. A key difference to MS junction is that a p-n diode contains two depletion regions of opposite type.The general analysis starts by setting up Poisson's equation:

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\varepsilon_s} = -\frac{q}{\varepsilon_s}(p - n + N_d^+ - N_a^-)$$

$$\frac{d^2\phi}{dx^2} = \frac{2qn_i}{\varepsilon_s}\left(\sinh\frac{\phi - \phi_F}{V_t} + \sinh\frac{\phi_F}{V_t}\right)$$

$$\sinh\frac{\phi_F}{V_t} = \frac{N_a^- - N_d^+}{2n_i}$$

This second-order non-linear differential equation cannot be solved analytically.

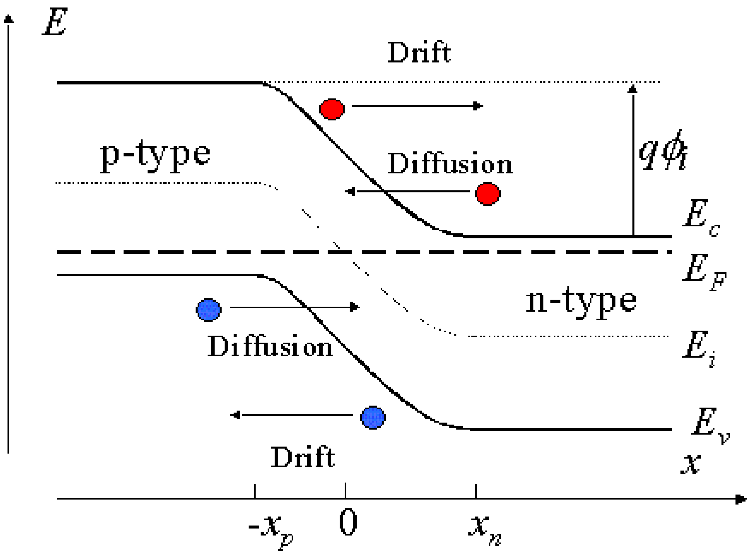
A simplifying assumption that the depletion region is fully depleted and that the adjacent neutral regions contain no charge is made, known as the full depletion approximation.

P-N junction: Full depletion approximation

The full-depletion approximation assumes that the depletion region around the junction has well-defined edges. It also assumes that the transition between the depleted and the quasi-neutral region is abrupt. We define the quasi-neutral region as the region adjacent to the depletion region where the electric field is small and the free carrier density is close to the net doping density.

The sum of the two depletion layer widths in each region is the total depletion layer width x_d:

$$x_d = x_n + x_p$$



The charge density profile is:

$$\rho = q(p - n + N_d^+ - N_a^-) \cong q(N_d^+ - N_a^-), \text{for } -x_p \le x \le x_n$$

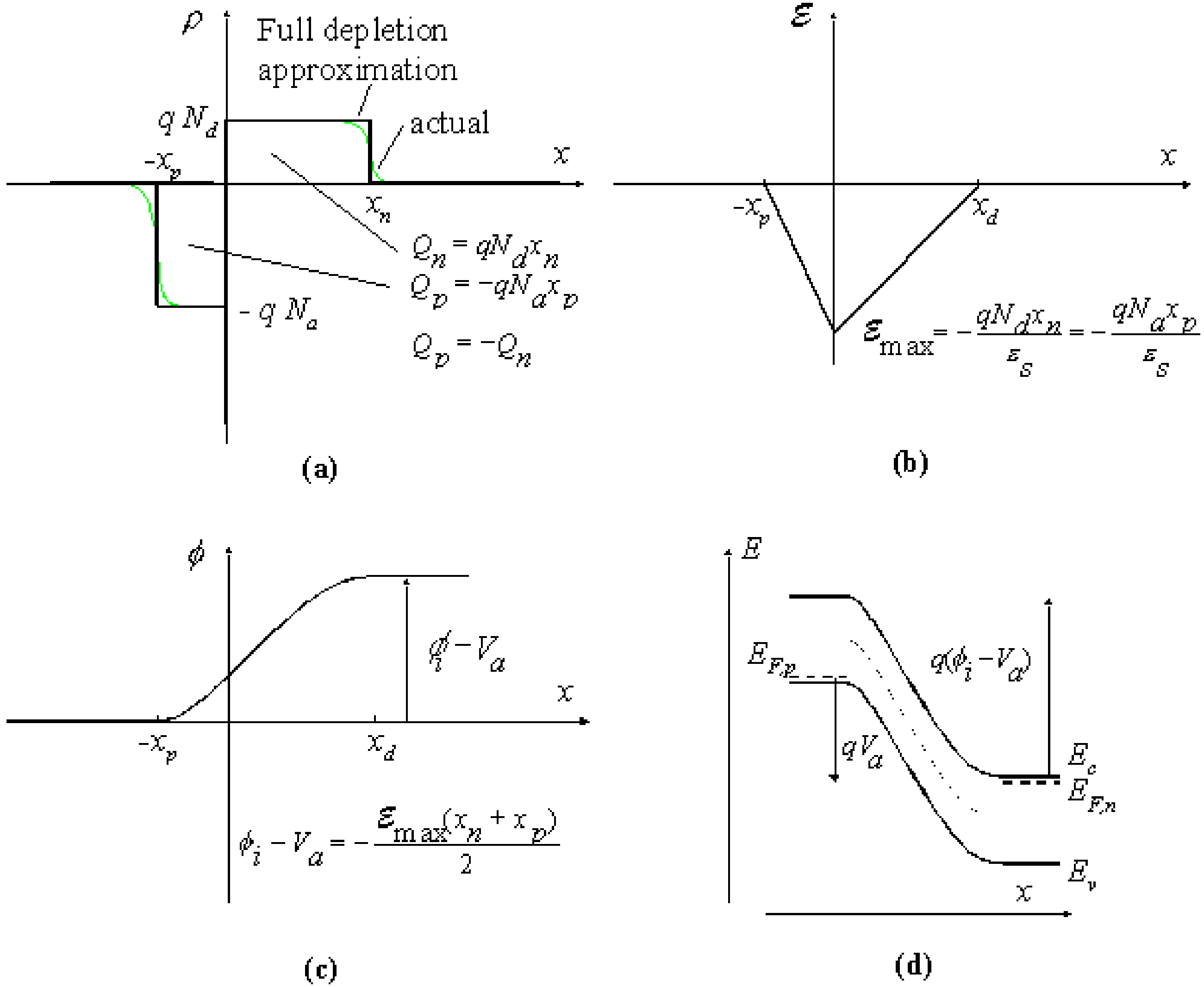
$$\rho(x) = 0$$

$$\rho(x) = -qN_a$$

$$\rho(x) = qN_d$$

$$\rho(x) = 0$$

P-N junction: Full depletion approximation



(a) Charge density in a p-n junction, (b) Electric field, (c) Potential and (d) Energy band diagram

P-N junction: Full depletion approximation

The charge in the n-type region, Q_n, and the charge in the p-type region, Q_p, are given by:

$$Q_n = qN_d x_n$$

$$Q_p = -qN_a x_p$$

Gauss's Law:

$$\frac{d\mathcal{E}(x)}{dx} = \frac{\rho(x)}{\varepsilon_s} \cong \frac{q}{\varepsilon_s}(N_d^+(x) - N_a^-(x)), \text{for } -x_p \le x \le x_n$$

$$\mathcal{E}(x) = 0$$

$$\mathcal{E}(x) = -\frac{qN_a(x + x_p)}{\varepsilon_s}$$

$$\mathcal{E}(x) = \frac{qN_d(x - x_n)}{\varepsilon_s}$$

$$\mathcal{E}(x) = 0$$

This maximum field can be calculated on either side of the depletion region, yielding:

$$\mathcal{E}(x = 0) = -\frac{qN_a x_p}{\varepsilon_s} = -\frac{qN_d x_n}{\varepsilon_s}$$

Relationship between 2 unknowns:

$$N_d x_n = N_a x_p$$

P-N junction: Full depletion approximation

$$x_d = x_n + x_p$$

$$x_n = x_d \frac{N_a}{N_a + N_d}$$

$$N_d x_n = N_a x_p$$

$$x_p = x_d \frac{N_d}{N_a + N_d}$$

The potential in the semiconductor is obtained from the electric field using: $\frac{d\phi(x)}{dx} = -\mathcal{E}(x)$

The total potential across the semiconductor must equal the difference between the built-in potential and the applied voltage, which provides a second relation between x_p and x_n:

$$\phi_i - V_a = \frac{qN_d x_n^2}{2\varepsilon_s} + \frac{qN_a x_p^2}{2\varepsilon_s}$$

$$x_d = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(\phi_i - V_a)}$$

$$x_n = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_a}{N_d}\frac{1}{N_a + N_d}(\phi_i - V_a)}$$

$$x_p = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_d}{N_a}\frac{1}{N_a + N_d}(\phi_i - V_a)}$$

P-N junction: Full depletion approximation

An abrupt silicon ($n_I = 10^{10}$ cm^{-3}) p-n junction consists of a p-type region containing 10^{16} cm^{-3} acceptors and an n-type region containing 5 x 10^{16} c cm^{-3} donors.

1. Calculate the built-in potential of this p-n junction.
2. Calculate the total width of the depletion region if the applied voltage V_a equals 0, 0.5 and -2.5 V.
3. Calculate maximum electric field in the depletion region at 0, 0.5 and -2.5 V.
4. Calculate the potential across the depletion region in the n-type semiconductor at 0, 0.5 and -2.5 V.

Solution

The built-in potential is calculated from: $\phi_i = V_t \ln \frac{p_n n_p}{n_i^2} = 0.0259 \ln \frac{10^{16} \times 5 \times 10^{16}}{10^{20}} = 0.76\ \text{V}$

The depletion layer width is obtained from: $x_d = \sqrt{\frac{2\varepsilon_s}{q}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)(\phi_i - V_a)}$

the electric field from $\mathcal{E}(x=0) = -\frac{2(\phi_i - V_a)}{x_d}$

and the potential across the n-type region equals $\phi_n = \frac{q N_d x_n^2}{2\varepsilon_s}$ where $x_n = x_d \frac{N_a}{N_a + N_d}$

$$\phi_n = \frac{(\phi_i - V_a) N_a}{N_a + N_d}$$

	$V_a = 0$ V	$V_a = 0.5$ V	$V_a = -2.5$ V
x_d	0.315 μm	0.143 μm	0.703 μm
$\mathcal{E}$	40 kV/cm	18 kV/cm	89 kV/cm
ϕ_n	0.105 V	0.0216 V	0.522 V