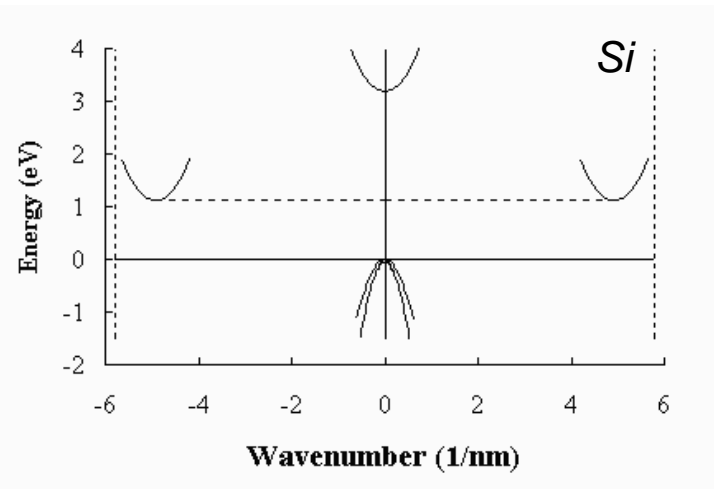
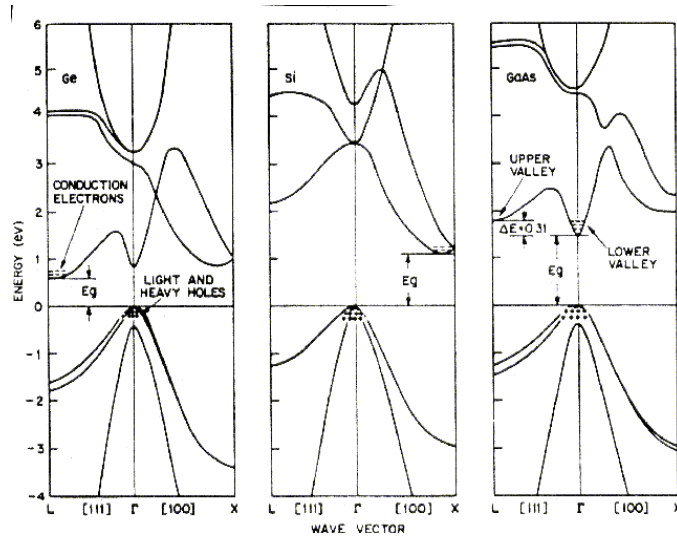
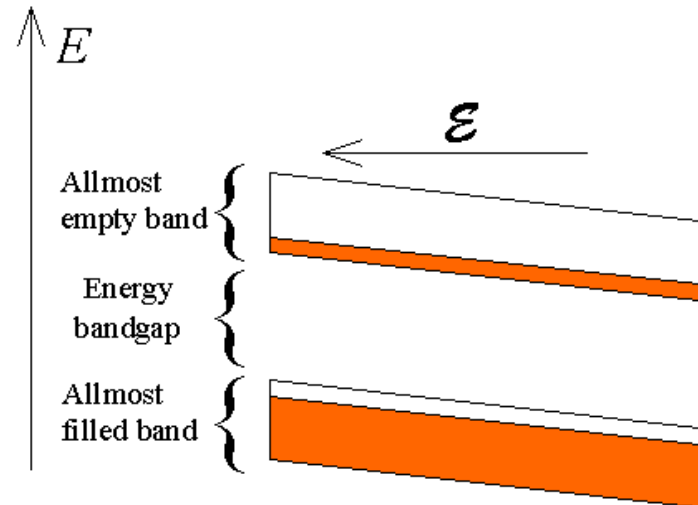
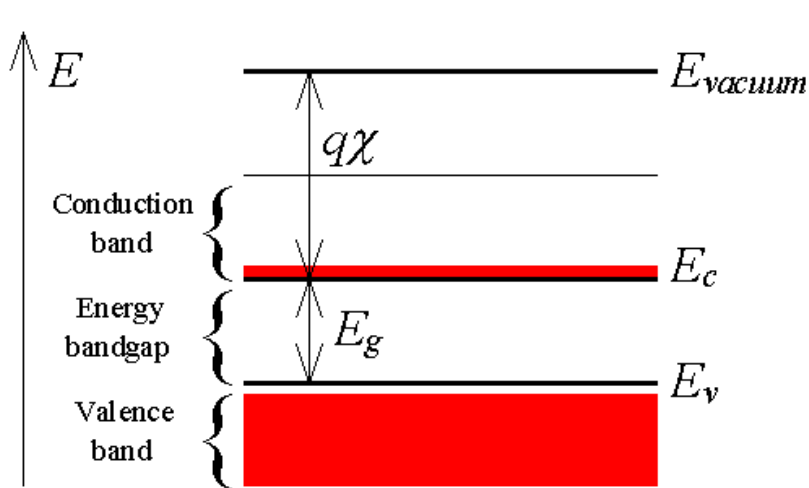


# Previous Lectures Review -- Band Diagrams

<http://sirius.ucsc.edu/sacarter/courses/>



*Simplify Band Diagrams by just considering bands near minimum energy gap*



# Previous Lectures Review -- Carrier Density

## Carrier Density

$$n(E) = g_c(E)f(E)$$

$$p(E) = g_v(E)[1 - f(E)]$$

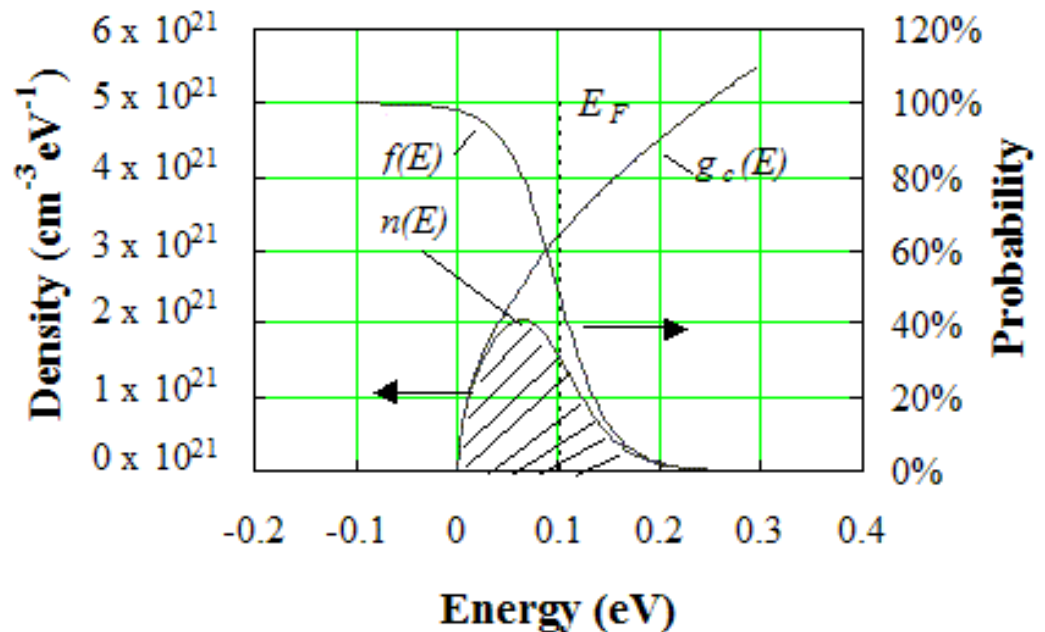
## Density of States

$$g_c(E) = \frac{8\pi\sqrt{2}}{h^3} m_e^{*3/2} \sqrt{E - E_c}, \text{ for } E \geq E_c$$

$$g_v(E) = \frac{8\pi\sqrt{2}}{h^3} m_h^{*3/2} \sqrt{E_v - E}, \text{ for } E \leq E_v$$

## Fermi-Dirac Distribution

$$f(E) = \frac{1}{1 + e^{(E - E_F)/kT}}$$



# Previous Lectures -- Intrinsic Carrier Density

At  $T=0$

$$n_o = \frac{2 \sqrt{2}}{3 \pi^2} \frac{m_e^{*3/2}}{\hbar^3} (E_F - E_c)^{3/2}, \text{ for } E_F \geq E_c$$

$$p_o = \frac{2 \sqrt{2}}{3 \pi^2} \frac{m_h^{*3/2}}{\hbar^3} (E_v - E_F)^{3/2}, \text{ for } E_v \geq E_F$$

*Intrinsic Carrier Density*

$$n_i = \sqrt{N_c N_v} e^{-E_g/2kT}$$

*Intrinsic Fermi Energy*

$$E_i = \frac{E_c + E_v}{2} + \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right)$$

*Where the effective density of states in conduction and valence band are:*

$$N_c = 2 \left[ \frac{2 \pi m_e^* kT}{h^2} \right]^{3/2}$$

$$N_v = 2 \left[ \frac{2 \pi m_h^* kT}{h^2} \right]^{3/2}$$

# Examples: Intrinsic Carrier Density

Example 2.4a Calculate the effective densities of states in the conduction and valence bands of germanium at 300 K.

$$\begin{aligned}N_c &= 2 \left( \frac{2 \pi m_e^* kT}{h^2} \right)^{3/2} \\&= 2 \left( \frac{2 \pi 0.55 \times 9.11 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.626 \times 10^{-34})^2} \right)^{3/2} \\&= 1.02 \times 10^{25} \text{ m}^{-3} = 1.02 \times 10^{19} \text{ cm}^{-3}\end{aligned}$$

Example 2.4b Calculate the effective carrier density of Si at 300 K.

$$\begin{aligned}n_i(300\text{K}) &= \sqrt{N_c N_v} \exp\left(\frac{-E_g}{2kT}\right) \\&= \sqrt{2.81 \times 10^{19} \times 1.83 \times 10^{19}} \exp\left(\frac{-1.12}{2 \times 0.0258}\right) \\&= 8.72 \times 10^9 \text{ cm}^{-3}\end{aligned}$$

# Previous Lectures Review -- Charge Transport

## Diffusion

$$J_n = qD_n \frac{dn}{dx}$$

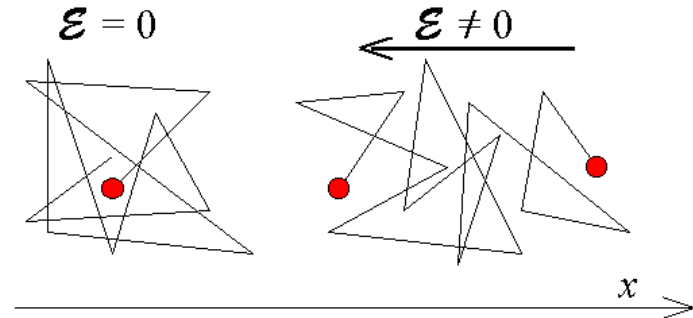
$$J_p = -qD_p \frac{dp}{dx}$$

where

$$D_n = \mu_n \frac{kT}{q} = \mu_n V_t \quad D_p = \mu_p \frac{kT}{q} = \mu_p V_t$$

$$v_{th} = \frac{l}{\tau}$$

$v_{th}$  is the thermal velocity



## Drift

$$\bar{J}_n = qn\mu_n \bar{\mathcal{E}}$$

$$\bar{J}_p = qp\mu_p \bar{\mathcal{E}}$$

$$\mu = \frac{q\tau_c}{m^*}$$

$\tau_c$  is time between collisions

$m^*$  is effective mass

**Total Current:**

$$J_n = qn\mu_n \mathcal{E} + qD_n \frac{dn}{dx}$$

$$J_p = qp\mu_p \mathcal{E} - qD_p \frac{dp}{dx}$$

# Conductivity and Resistivity

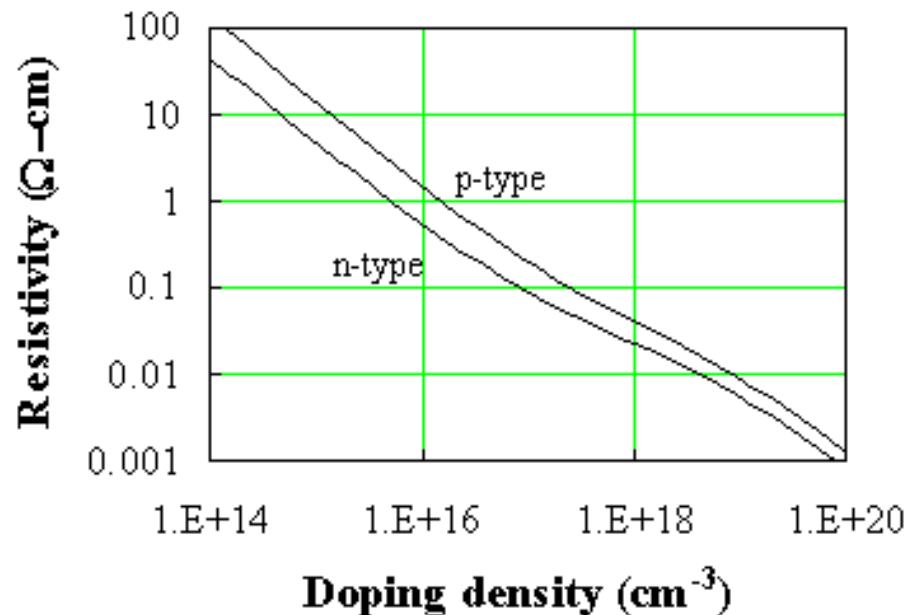
The conductivity  $\sigma$  of a material is the current density divided by the applied electric field.

$$J = qnv_e + qp v_h = q(n\mu_n + p\mu_p)\mathcal{E}$$

$$\sigma = \frac{\Delta J}{\mathcal{E}} = q(n\mu_n + p\mu_p)$$

The resistivity  $\rho$  is defined as the inverse of the conductivity

$$\rho = \frac{1}{\sigma} = \frac{1}{q(\mu_n n + \mu_p p)}$$



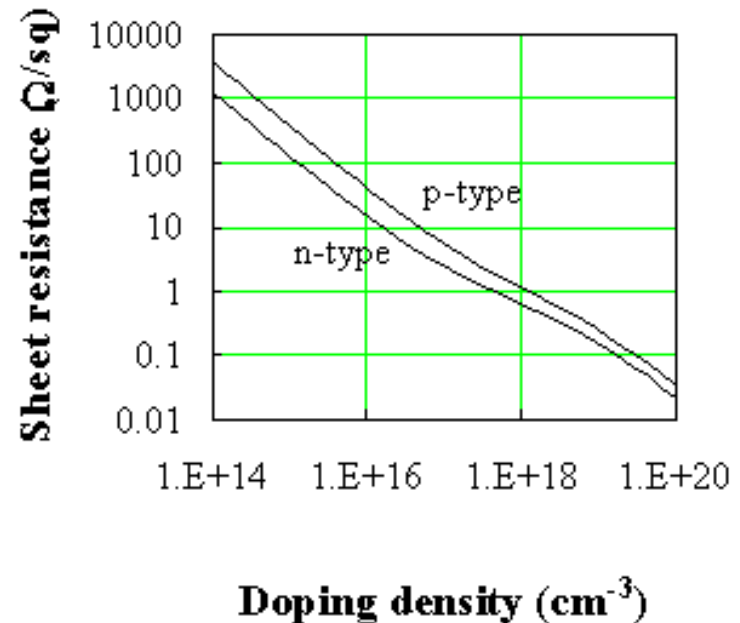
# Sheet Resistance

Sheet Resistance  $R_s$  is a useful quantity when dealing with thin films. The sheet resistance of a uniformly-doped layer with resistivity,  $\rho$ , and thickness,  $t$ , is given by:

$$R_s = \frac{\rho}{t}$$

While the unit of the sheet resistance is Ohms, one refers to it as Ohms per square. This nomenclature is useful for the resistance of a rectangular piece of material with length,  $L$ , and width  $W$  must be obtained.

$$R = R_s \frac{L}{W}$$



# Sheet Resistance Example

Example 2.9 A piece of silicon doped with arsenic ( $N_d = 10^{17} \text{ cm}^{-3}$ ) is 100 mm long, 10 mm wide and 1 mm thick. Calculate the resistance of this sample when contacted one each end.

$$\rho = \frac{1}{qn\mu_n} = \frac{1}{1.6 \times 10^{-19} \times 10^{17} \times 727} = 0.086 \text{ } \Omega\text{cm}$$

$$R = \rho \frac{L}{Wt} = 0.086 \times \frac{100 \times 10^{-4}}{10 \times 10^{-4} \times 10^{-4}} = 8.6 \text{ k}\Omega$$

$$R_s = \frac{\rho}{t} = \frac{0.086}{10^{-4}} = 860 \text{ } \Omega / \text{square}$$

$$R = R_s \frac{L}{W} = 860 \times \frac{100 \times 10^{-4}}{10 \times 10^{-4}} = 8.6 \text{ k}\Omega$$

# Carrier Recombination and Generation

Recombination of electrons and holes is a process by which both carriers annihilate each other: electrons occupy the empty state associated with a hole. Both carriers eventually disappear in the process and the energy difference between the initial and final state of the electron is released.

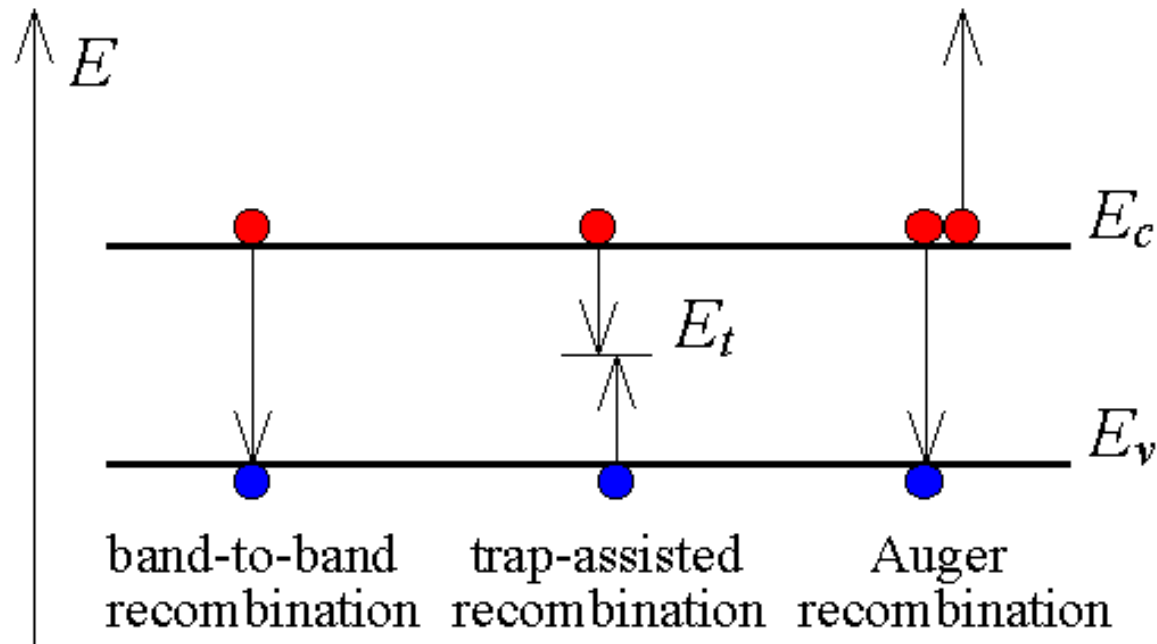
$$E_{\text{released}} = E_2 - E_1$$

*Radiative recombination -- Photon is released*

*Non-radiative recombination -- Phonon is released*

*Auger recombination -- Kinetic energy is released (given off to another electron)*

# Carrier Recombination



Band-to-band: an electron moves from its conduction band state into the empty valence band state associated with the hole (mechanism for radiative recombination).

Trap-assisted recombination: an electron falls into a "trap", an energy level within the bandgap caused by the presence of a foreign atom or a structural defect (Shockley-Hall-Read)

Auger recombination: an electron and a hole recombine in a band-to-band transition, but now the resulting energy is given off to another electron or hole.

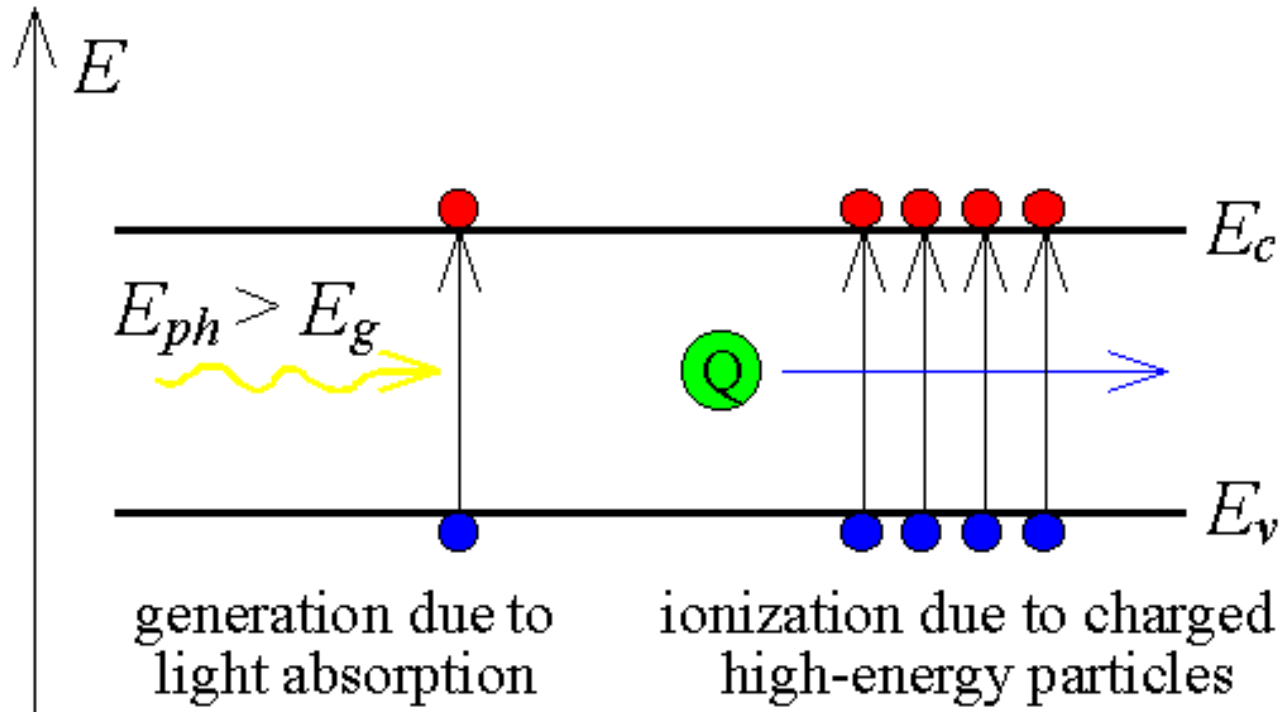
# Carrier Generation

Each carrier recombination method can be reversed to result in carrier generation; therefore, a single expression can frequently be used to describe both carrier generation and recombination.

Other Generation Mechanisms:

- Photon (light) absorption (related to band-to-band recombination)
- High energy electron or particle beams
- Impact Ionization (related to Auger recombination)
- Direct charge injection

# Carrier Generation

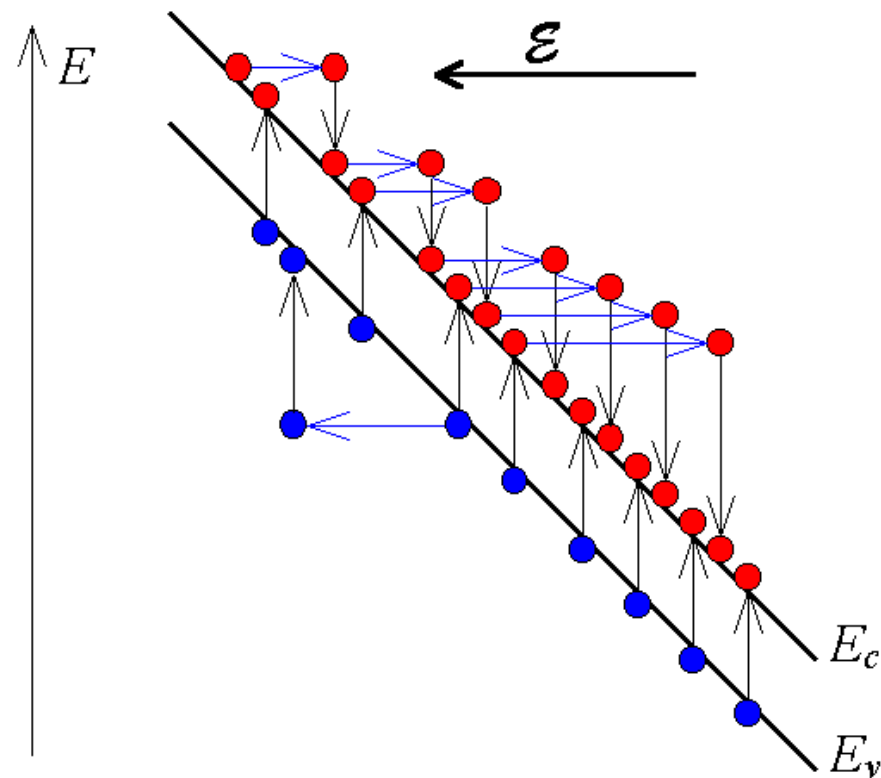


*Photon absorbed and excess energy is added to electron to excite it to conduction band*

*Ionization can result in available energy much larger than the bandgap energy so that multiple electron-hole pairs can be formed.*

# Carrier Generation: Impact Ionization

In Impact Ionization, the excess energy is given off to generate an electron-hole pair through a band-to-band transition. This generation process causes avalanche multiplication in semiconductor diodes under high reverse bias. The kinetic energy is given off to an electron in the valence band, thereby creating an electron-hole pair. The resulting two electrons can create two more electrons which generate four more causing an avalanche multiplication effect. Electrons as well as holes contribute to avalanche multiplication.



# Simple Recombination-Generation Model

The recombination {R} minus the Generation {G} rate, resulting in a net rate {U}, is proportional to the excess carrier density. No net recombination takes place if the carrier density equals the thermal equilibrium value.

*For electrons in a p-type semiconductor:* 
$$U_n = R_n - G_n = \frac{n_p - n_{p0}}{\tau_n}$$

*For holes in a n-type semiconductor:* 
$$U_p = R_p - G_p = \frac{p_n - p_{n0}}{\tau_p}$$

where  $\tau$  is the average time after which an excess minority carrier recombines.

The recombination rate of the **majority** carriers depends on the excess-**minority**-carrier-density as the *minority carriers limit the recombination rate*.

# Band-to-Band

Band-to-band recombination depends on the density of available electrons and holes. Both carrier types need to be available in the recombination process. Therefore, the rate is expected to be proportional to the product of n and p.

In thermal equilibrium, the recombination rate must equal the generation rate since there is no net recombination or generation. As the product of n and p equals  $n_i^2$  in thermal equilibrium, the net recombination rate can be expressed as:

$$U_{b-b} = b(np - n_i^2)$$

where b is the bimolecular recombination constant.

# Trap-assisted (Shockley-Hall-Read)

*Complicated Derivation!*

$$U_{SHR} = \frac{pn - n_i^2}{p + n + 2n_i \cosh\left(\frac{E_i - E_t}{kT}\right)} N_t v_{th} \sigma$$

If  $p \gg n$ , then  $U_n = R_n - G_n = \frac{n_p - n_{p0}}{\tau_n}$

If  $n \gg p$ , then  $U_p = R_p - G_p = \frac{p_n - p_{n0}}{\tau_p}$

where  $\tau_n = \tau_p = \frac{1}{N_t v_{th} \sigma}$

$N_t$  is density of traps,  $v_{th}$  is the thermal velocity, and  $\sigma$  is the conductivity

# Surface Recombination

Surfaces and interfaces typically contain a large number of recombination centers because of the abrupt termination of the semiconductor crystal, which leaves a large number of electrically active states. In addition, the surfaces and interfaces are more likely to contain impurities since they are exposed during the device fabrication process. The net recombination rate due to trap-assisted recombination and generation is given by:

$$U_{s,SHR} = \frac{pn - n_i^2}{p + n + 2n_i \cosh\left(\frac{E_i - E_{st}}{kT}\right)} N_{st} v_{th} \sigma_s$$

If  $E_i = E_{st}$

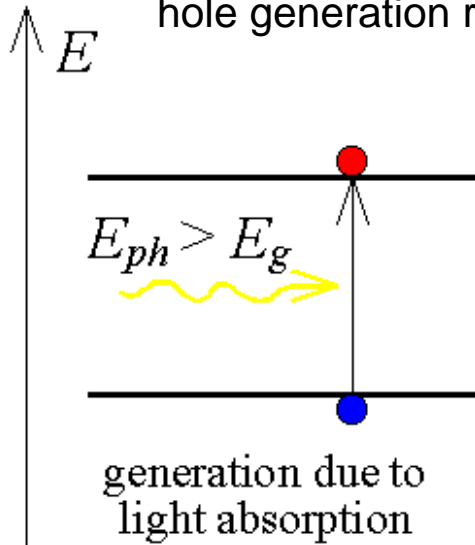
$$U_{s,n} = R_{s,n} - G_{s,n} = v_s (n_p - n_{p0})$$

*Recombination velocity*  $v_s = N_{st} v_{th} \sigma_s$

$N_{st}$  is density of surface traps and  $E_{st}$  is energy of surface

# Generation due to Light

If each absorbed photon creates one electron-hole pair, the electron and hole generation rates are given by:



$$G_{p,light} = G_{n,light} = \alpha \frac{P_{opt}(x)}{E_{ph}A}$$

where  $\alpha$  is the absorption coefficient of the material at the energy of the incoming photon. The absorption of light causes the optical power  $P_{opt}$  to decrease with distance.

$$\frac{dP_{opt}(x)}{dx} = -\alpha P_{opt}(x)$$

The excess carriers generated  $\delta n = \delta p = \tau_p G_p$

# Generation due to Light: Example

Example 2.11 Calculate the excess electron and hole densities in an n-type silicon wafer ( $N_d = 10^{17} \text{ cm}^{-3}$ ) illuminated uniformly with  $10 \text{ mW/cm}^2$  of red light ( $E_{ph} = 1.8 \text{ eV}$ ). The absorption coefficient of red light in silicon is  $10^{-3} \text{ cm}^{-1}$ . The minority carrier lifetime is  $10 \text{ ms}$ .

The generation rate of electrons and holes equals:

$$G_n = G_p = \alpha \frac{P_{opt}}{E_{ph} A} = 10^{-3} \frac{10^{-2}}{1.8 \times 1.6 \times 10^{-19}} = 3.5 \times 10^{13} \text{ cm}^{-3} \text{ s}^{-1}$$

where the photon energy was converted into Joules. The excess carrier densities are then obtained from:

$$\delta n = \delta p = \tau_p G_p = 10 \times 10^{-6} \times 3.5 \times 10^{13} = 3.5 \times 10^8 \text{ cm}^{-3}$$

# Semiconductor Thermodynamics

The thermodynamic identity: adding heat, work or particles can cause a change in energy.

$$dU = dQ + dW + \mu dN$$

where  $U$  is the energy,  $Q$  is the heat and  $W$  is the work.  $\mu$  is the energy added to a system when adding one particle without adding either heat or work. The amount of exchanged heat depends on the temperature,  $T$ , and the entropy,  $S$ , while the amount of work delivered to a system depends on the pressure,  $p$ , and the volume,  $V$ , or:

$$dQ = TdS$$

$$dW = -pdV$$

$$dU = TdS - pdV + \mu dN$$

# The Fermi Energy

The Fermi energy,  $E_F$ , is the energy associated with a particle, which is in thermal equilibrium with the system of interest. The energy is strictly associated with the particle and does not consist of heat or work. This same quantity is called the electrochemical potential,  $\mu$ .

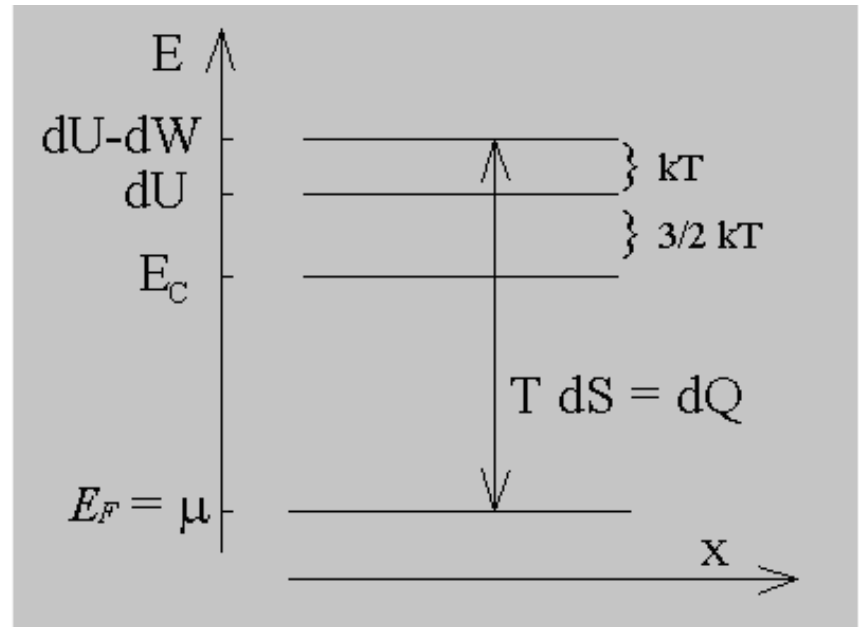
$$dU = \cancel{dQ} + \cancel{dW} + \mu dN$$

$$U = \frac{3}{2} NkT + NE_c$$

$$E_F = E_c + kT \ln \frac{n}{N_c}$$

$$S = \frac{U - \mu N + pV}{T}$$

$$S = \frac{5}{2} Nk - Nk \ln \frac{n}{N_c}$$



Energy, work and heat per electron in an ideal electron gas visualized on an energy band diagram.

# Quasi Fermi Energy

Quasi-Fermi energies are introduced when the electrons and holes are clearly not in thermal equilibrium with each other. Even though electrons and holes are not in thermal equilibrium, they still are in thermal equilibrium with themselves and can still be described by a Fermi energy which is now different for the electrons and the holes. These Fermi energies are referred to as the electron and hole quasi-Fermi energies,  $F_n$  and  $F_p$ .

$$n = n_i \exp \frac{F_n - E_i}{kT} = N_c \exp \frac{F_n - E_c}{kT}$$

$$p = n_i \exp \frac{E_i - F_p}{kT} = N_v \exp \frac{E_v - F_p}{kT}$$

The energy loss in a recombination process equals the difference between the electron and hole quasi-Fermi energies:

$$\Delta U = F_n - F_p$$

No heat or work is removed from the system, just the energy associated with the particles. The energy lost in the recombination process can be converted in heat or light depending on the details of the process.

# The Drift-Diffusion Model

The assumptions of the simplified drift-diffusion model are:

Full ionization: all dopants are assumed to be ionized (shallow dopants)

Non-degenerate: the Fermi energy is assumed to be at least  $3 kT$  below/above the conduction/valence band edge.

Steady state: All variables are independent of time.

Constant temperature: The temperature is constant throughout the device.

The ten variables are the following:

$\rho$ , the charge density

$n$ , the electron density

$p$ , the hole density

$\varepsilon$ , the electric field

$\phi$ , the potential

$E_i$ , the intrinsic energy

$F_n$ , the electron quasi-Fermi energy

$F_p$ , the hole quasi-Fermi energy

$J_n$ , the electron current density

$J_p$ , the hole current density

# The Drift-Diffusion Model

Charge Density Equation  $\rho = q(p - n + N_d^+ - N_a^-)$

Electric Field and Potential Equations  $\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$        $\frac{d\phi}{dx} = -\mathcal{E}$        $\frac{dE_i}{dx} = q\mathcal{E}$

Carrier Density Equations  $n = n_i e^{(F_n - E_i)/kT}$        $p = n_i e^{(E_i - F_p)/kT}$

Drift and Diffusion Equations  $J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$        $J_p = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx}$

Continuity Equation in Steady State  
With SHR Recombination

$$0 = -\frac{1}{q} \frac{\partial J_p}{\partial x} - \frac{np - n_i^2}{n + p + 2n_i \cosh\left(\frac{E_t - E_i}{kT}\right)} \frac{1}{\tau}$$

$$0 = \frac{1}{q} \frac{\partial J_n}{\partial x} - \frac{np - n_i^2}{n + p + 2n_i \cosh\left(\frac{E_t - E_i}{kT}\right)} \frac{1}{\tau}$$

# Thermoelectric Effects in Semiconductors

The temperature dependence of the current in a semiconductor can be included by generalizing the drift-diffusion current equation. The proportionality constant between the current density and the temperature gradient is the product of the conductivity,  $\sigma$ , and the thermoelectric power,  $P$  or  $\Pi$ .

$$J_n = q\mu_n n \mathcal{E} + q \frac{d(D_n n)}{dx} \qquad n = N_c \exp \frac{E_F - E_c}{kT}$$

yields 
$$J_n = \mu_n n \left( \frac{dE_c}{dx} + k \frac{dT}{dx} + \frac{kT}{\mu_n} \frac{d\mu_n}{dx} + \frac{kT}{N_c} \frac{dN_c}{dx} + kT \frac{d\left(\frac{E_F - E_c}{kT}\right)}{dx} \right)$$

For the case where the material properties do not change with position, all the spatial variations except for the gradient of the Fermi energy are caused by a temperature variation. The current can be written in the following form:

$$J_n = \mu_n n \left( \frac{dE_c}{dx} - qP \frac{dT}{dx} \right) \qquad \text{and } P \text{ is the thermoelectric power in Volt/Kelvin.}$$

# Thermoelectric Effects in Semiconductors

$$\mathcal{P}_n = -\frac{k}{q} \left( \frac{5}{2} + \frac{T}{\mu_n} \frac{d\mu_n}{dT} + \ln \frac{N_c}{n} \right)$$

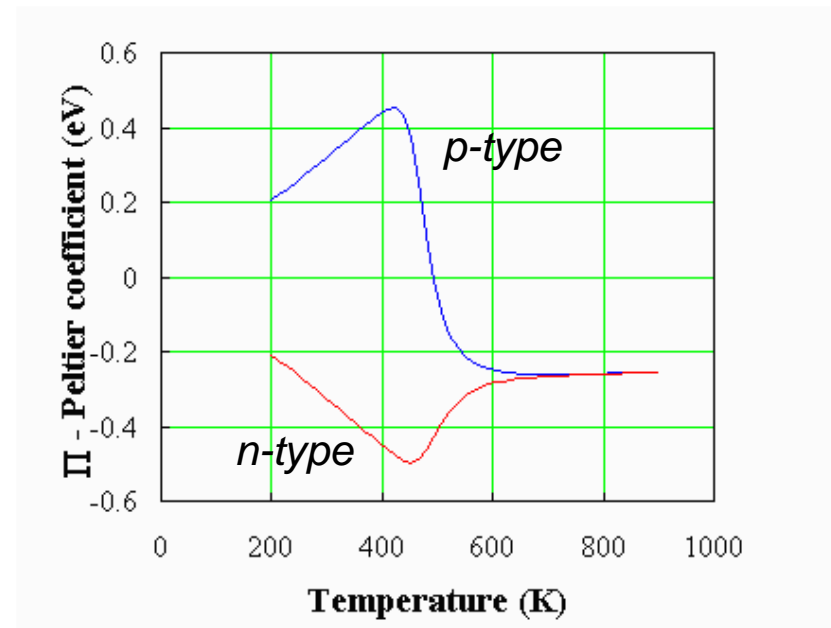
If the temperature dependence of the mobility is expressed as a simple power law:  $\mu_n \propto T^{-s}$

then

$$\mathcal{P}_n = -\frac{k}{q} \left( \frac{5}{2} - s + \ln \frac{N_c}{n} \right) \qquad \mathcal{P}_p = \frac{k}{q} \left( \frac{5}{2} - s + \ln \frac{N_v}{p} \right)$$

The Peltier coefficient,  $\Pi$ , is related to the thermoelectric power by:

$$\Pi = \mathcal{P}T$$

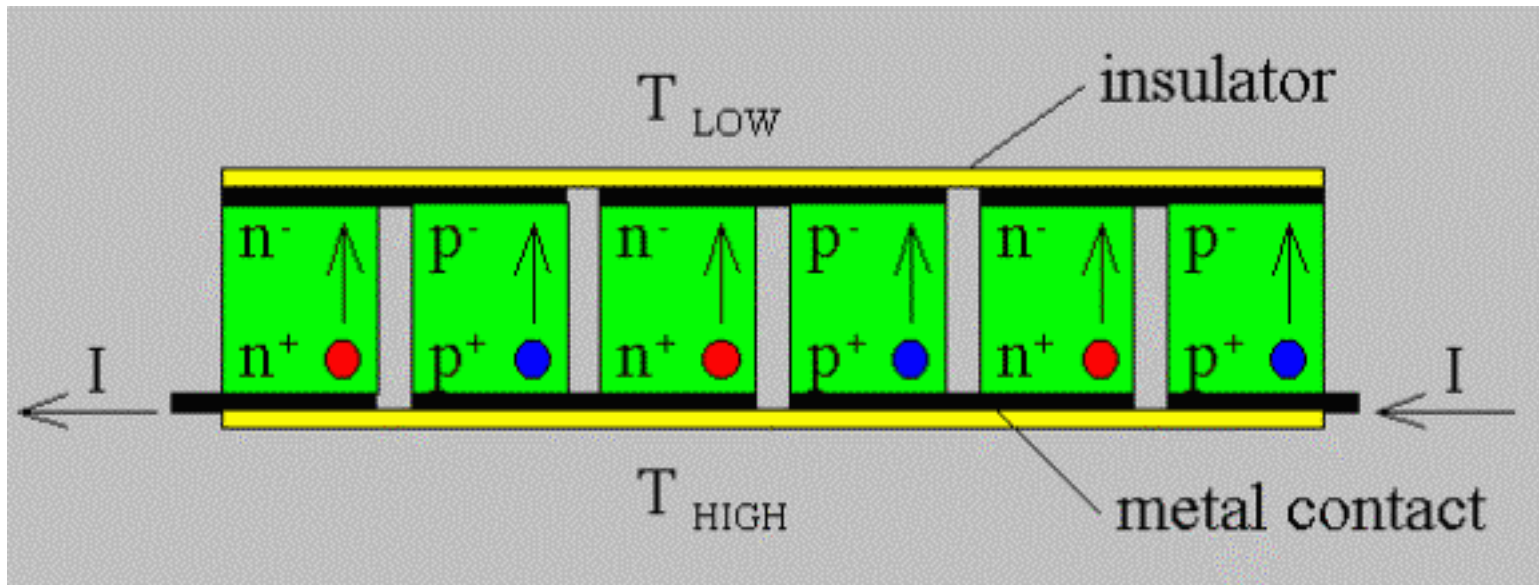


# The Thermoelectric Cooler

Thermoelectric effects in semiconductors cause carriers to flow due to temperature gradients but also cause temperature gradients when an electrical current is applied. The thermoelectric cooler is a device in which a current is applied to a semiconductor causing a temperature reduction and cooling.

*Applications: cooling chips and/or car seats, power generation*

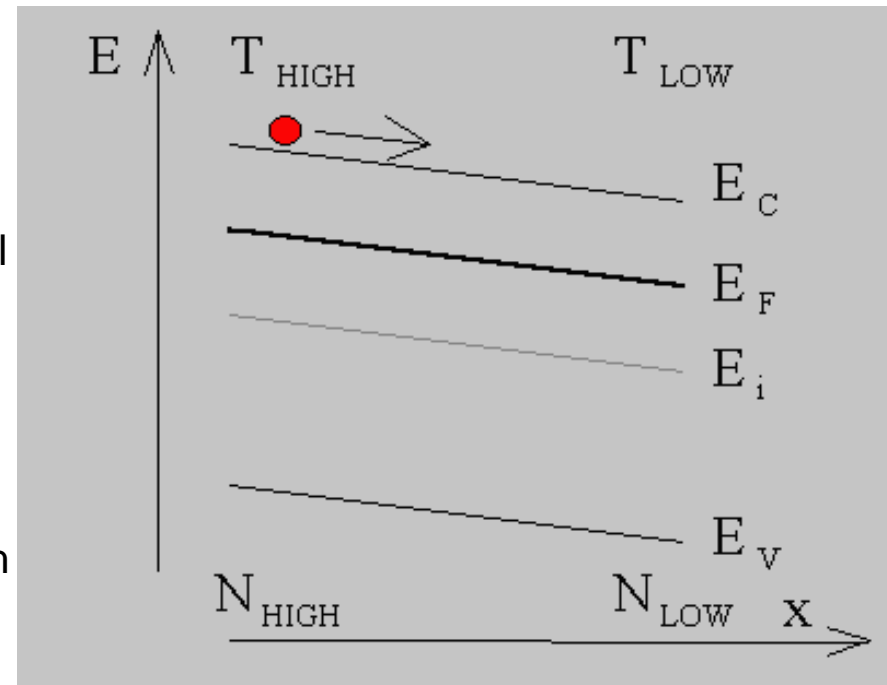
Cross-section of a thermoelectric cooler showing the alternating n-type and p-type sections. The series connection avoids the high current requirement of the single element.



The operation of the thermoelectric cooler is similar to that of a Joule-Thomson refrigerator in that an expansion of a gas is used to cool it down. While heating of a gas can be obtained by compressing it as is the case in a bicycle, a gas can also be cooled by expanding it into a larger volume.

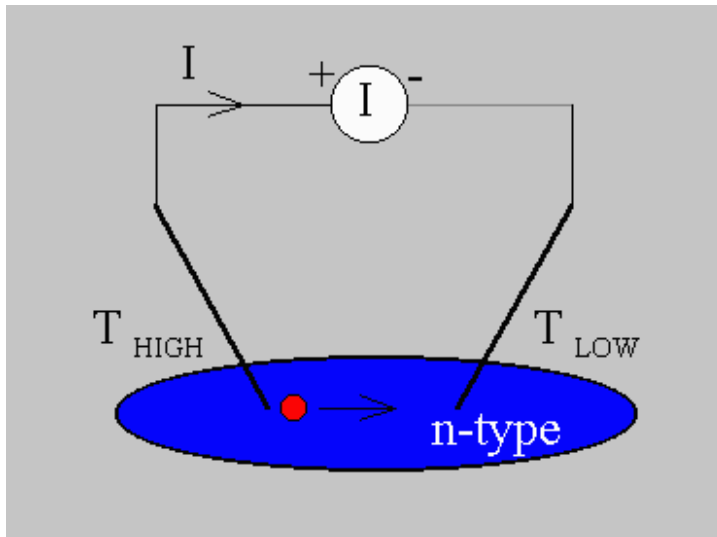
In a thermoelectric cooler, a current is applied such that carriers flow from the high density (high  $T$ ) region to the low-density (low  $T$ ) region, the volume around a fixed number of carriers must increase as the carriers move towards the lower doped region. At constant  $T$  and in thermal equilibrium there is no current as the diffusion current is balanced by the drift current associated with the built-in electric field caused by the graded doping density. As a current is applied to the semiconductor the built-in field is reduced so that the carriers diffuse from the high to low doping density. This causes a  $T$ -reduction on the low-doped side, which continues until the entropy is constant throughout the semiconductor.

## The Thermoelectric Cooler

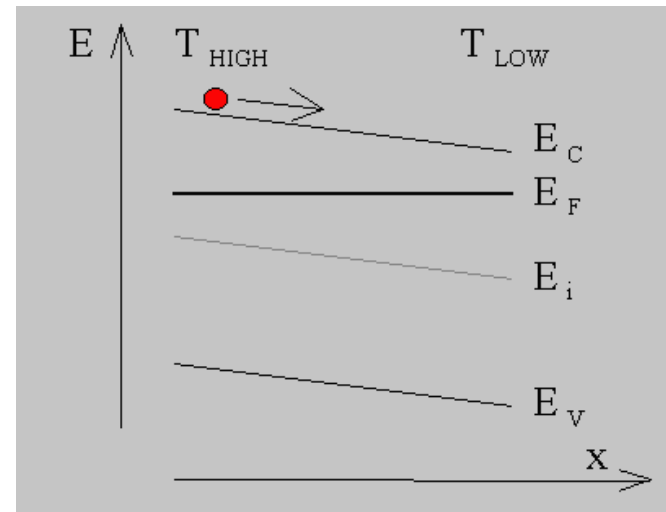


# The Hot Probe Experiment

The "hot-probe" experiment provides a very simple way to distinguish between n-type and p-type semiconductors using a soldering iron and a standard multimeter.



When applying the probes to n-type material one obtains a positive current reading on the meter, while p-type material yields a negative current.



As the effective density of states decreases with decreasing  $T$ ,  $E_c$  decreases with decreasing  $T$  yielding an electric field, which causes the electrons to flow from the high to the low  $T$ .

$$J_n = \mu_n n \left( \frac{dE_c}{dx} - q \mathcal{P} \frac{dT}{dx} \right)$$